Gauge-Higgs unification
と階層性問題
〜初心者のための入門〜

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1. The hierarchy problem

Though the standard model (SM) is successful, from theory point of view it is not completely satisfying:

- gravity is not included
- not a real unified theory
- too many arbitrary parameters (the origin of flavor?)

Thus, SM is expected to be replaced by more fundamental theory at higher energy.

Namely SM is an effective low energy theory, valid up to $\Lambda$, physical cutoff.

The hierarchy problem:

$\Lambda \sim M_{GUT}, M_{pl}$

how to maintain $M_W \ll \Lambda$
Since,

\[ m_H^2 \sim \lambda v^2, \quad N_W \sim g v, \quad \rightarrow m_H \sim M_W \]

the problem is equivalent to ask how the Higgs mass is kept small (at weak scale).

Two kinds of hierarchy problems:

(1) @ classical level

Take SU(5) grand unified theory (GUT), as an example of fundamental theory.

It unifies SU(3)xSU2)xU(1) interactions in SU(5).

At the same time it unifies quarks & leptons:

\[
\begin{pmatrix}
  d^r \\
  d^b \\
  d^g \\
  e^+ \\
  \bar{\nu}_e
\end{pmatrix}
\]
Similarly, Higgs doublet is accompanied by colored Higgs:

\[ \begin{pmatrix} h^r \\ h^b \\ h^g \\ h^+ \\ h^0 \end{pmatrix} \]

The exotic colored Higgs should be heavy with the mass of the order \( M_{GUT} \) to avoid too rapid proton decay. 

\[ \text{"triplet-doublet splitting problem"} \]

In order to realize light doublet, while keeping the colored Higgs heavy, fine tuning of scalar potential with accuracy \( \frac{M_W^2}{M_{GUT}^2} \sim 10^{-26} \) is necessary \( \rightarrow \) unnatural
(Scalar potential)

\[ V(H, \Sigma) = -\frac{1}{2} \nu^2 H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 + \alpha (H^\dagger H) \text{Tr}(\Sigma^2) + \beta H^\dagger \sigma^2 H \]

\[ H, \Sigma : 5, 24 \text{ repr. of SU}(5) \]

After \( \Sigma \) developing its VEV of

\[ \langle \Sigma \rangle = V \cdot \text{diag}(1, 1, 1, -\frac{2}{3}, -\frac{2}{3}) \]

SU(5) is broken into SU(3)xSU(2)xU(1) \( \rightarrow V = \mathcal{O}(M_{GUT}) \)

For \( H \) the potential behaves as

\[ V(H, \Sigma) \sim -\frac{1}{2} \nu^2 H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 + \left( \frac{15}{2} \alpha + \frac{9}{4} \beta \right) V^2 (H^\dagger H) \]

\( \nu \) should be of \( \mathcal{O}(M_{GUT}) \) and the parameters \( \alpha, \beta \) should be fine-tuned at the order of \( \frac{M_W^2}{M_{GUT}^2} \sim 10^{-26} \)
(2) @ quantum level

Even if \( m_H \) is kept small at tree level, at quantum level, it gets large correction and fine tuning is necessary at all orders of perturbation theory.

\[
\lambda \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \propto \lambda \int_0^\Lambda dk \sim \lambda \Lambda^2 \quad (\equiv \delta m_H^2)
\]

: problem of “quadratic divergence”

\[
m_0^2 + \delta_H^2 = m_H^2 \quad (m_0: \text{bare Higgs mass})
\]

The bare mass-squares should be fine tuned at the precision

\[
\frac{M_W^2}{\Lambda^2} \sim 10^{-26} \quad \text{for} \quad \Lambda \sim M_{GUT}
\]

\( m_H \) is quite UV-sensitive!
To make it UV-insensitive, the “naturalness” condition (by ’t Hooft) is useful:

Some quantity may be naturally small provided the symmetry of the theory is enhanced by switching off the quantity.

(N.B.) If the condition is met, the quantum correction should be proportional to itself (bare parameter), not the cutoff. → UV-insensitive.

(example) fermion mass

\[ m \bar{\psi} \psi = m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \]

\[ m = 0 \quad \rightarrow \quad \text{chiral symmetry under the transformation} \]

\[ \psi \quad \rightarrow \quad \psi' = e^{i\varphi \gamma^5} \psi \]
e.g. in QED

\[ e_L \rightarrow e_L \rightarrow e_R \rightarrow e_R \sim \alpha \ m_e \ \log \Lambda \]
(no quadratic divergence)

2. Possible solutions in 4D based on symmetries

- Supersymmetry (SUSY)
- Dynamical symmetry breaking (Technicolor)
- Little Higgs (Higgs as a pseudo-Nambu-Goldstone boson)

For instance, in SUSY the mass of “higgsino” (the partner of Higgs) is protected by the chiral symmetry. Then SUSY guarantees the small \( m_H \), as long as SUSY breaking mass \( M_{SUSY} \) is not much larger than \( M_W \).
Or, quadratic divergence cancels out between boson and fermion loops:

\[
\begin{array}{c}
\hline
\text{h} \\
\hline
\end{array}
\leftarrow \begin{array}{c}
-i\lambda \\
\hline
F_h \\
\hline
\end{array}
\rightarrow \begin{array}{c}
-i\lambda \\
\hline
\text{h} \\
\hline
\end{array}
+ \\
\begin{array}{c}
\hline
\text{h} \\
\hline
\end{array}
\leftarrow \begin{array}{c}
-i\lambda \\
\hline
\psi_h \\
\hline
\end{array}
\rightarrow \begin{array}{c}
-i\lambda \\
\hline
\text{h} \\
\hline
\end{array}

\lambda^2\Lambda^2 + -\lambda^2\Lambda^2 = 0

(\psi_h : \text{higgsino}, \ F_h : \text{auxiliary field})

(N.B.)

“Little Higgs” utilizes global symmetry to keep Higgs mass small.

It is interesting that it has a close relation to Gauge-Higgs unification scenario, discussed later (work in progress w./ N.Kurahashi & K. Tanabe).
3. Possible solutions in the theories with extra dimensions

- Large extra space (Arkani-Hamed, Dimopoulos & Dvali (’98))
  Gravity itself in higher dimensional space-time is as “strong” as weak interaction, though 4D gravity is much weaker than gravity. How is such thing possible? → Matters and gauge bosons are Confined in a “brane”, while graviton can propagate in the “bulk”.

- Warped extra dimension (Randall and Sundrum (’99))
  5D Anti-de Sitter Space-time → corresponding to “inflation” through Wick rotation → “warp factor”
  \[ e^{-\kappa r_c} \quad (\kappa \sim M_{pl}, \quad r_c \sim l_{pl}, \quad M_W \sim e^{-\kappa r_c} M_{pl}) \]
  These scenarios, however, do not invoke any symmetry.
4. A solution based on gauge-Higgs unification


unification of gravity (s=2) & elemag (s=1) (A. Einstein)

Kaluza-Klein theory (higher dimensional gravity theory)

unified theory of gauge (s=1) & Higgs (s=0) interactions

“Gauge-Higgs unification”

: realized in higher dimensional gauge theory

Our idea: We regard the Higgs as a gauge boson !
And we invoke (higher dimensional) gauge symmetry to solve the hierarchy problem
(N.B.) Photon never gets a mass !

But, how the gauge boson with spin can be scalar particle ?
the idea of gauge-Higgs unification itself is not new:

- Y. Hosotani, Phys. Lett. B126 (’83) 309:

  S.S.B. due to \( \langle A_y^{(0)} \rangle \neq 0 \) : Hosotani mechanism
We may also regard the Higgs as a partner of 4D gauge boson, whose mass is protected by local gauge symmetry:

by enlarging 4D Poincaré symmetry,

\[ A_y' = A_y + \partial_y \lambda(y) \quad (D=5, \ U(1)) \]

\[ A_y \text{ transforms inhomogeneously} \]

(local) operator \( m^2 A_y^2 \) : forbidden by this gauge symmetry

We may also regard the Higgs as a partner of 4D gauge boson, whose mass is protected by local gauge symmetry:

by enlarging 4D Poincaré symmetry,

\[ 4D \text{ space-time} \iff \text{ 5D space-time} \]

\[ x^\mu \rightarrow x^M = (x^\mu, y) \]

(h, A_\mu)

(N.B.) in SUSY, 4D space-time \( \rightarrow \) superspace

\[ x^\mu \rightarrow (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) \]

(h, \psi_h)

s=0 s=1/2
importance of K-K (Kaluza-Klein) mode sum (@ quantum level)

A general wisdom: momentum cutoff spoils local gauge symmetry

(5D with extra-space S¹)

On mass-shell condition:
\[ p^M p_M - m^2 = p^\mu p_\mu - p_y^2 - m^2 = 0 \]  \( m \) : bulk mass

\[ \implies m_n^2 = \left(\frac{n}{R}\right)^2 + m^2 \]

Fields are K-K mode expanded (Fourier expansion):
\[ \phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \phi(n)(x^\mu) \cdot e^{in \frac{y}{R}} \]

Thus, to keep all (not only n=0) K-K (massive) modes:

essential to get finite \( m_H^2 \)
@ quantum level we get finite, but non-vanishing $m_H^2$

doesn’t it contradict with the “naturalness” argument?

we have seen the local operator

$$m^2 A_y^2$$

is forbidden by gauge symmetry

It turns out the effective potential of Wilson-loop, which is non-local gauge invariant operator, is induced at quantum level!

$$V_{\text{eff}}(W), \quad W = e^{ie \int A_y dy} = e^{i(2\pi Re A_y^{(0)})}$$

Since it’s non-local $W$ has nothing to do with UV-divergence, and the potential is completely finite.
<W> and <A^{(0)}_y> is fixed by minimization of V_{eff}(W)
(N.B.) non-vanishing <W> leads to S.S.B. for non-Abelian case:

**Hosotani-mechanism**

the quadratic term of A^{(0)}_y - <A^{(0)}_y> in V_{eff} provides the finite m_H^2

Is A^{(0)}_y, being just a constant, physical? Not pure-gauge?

Yes, \[ W = e^{i(2\pi R e A^{(0)}_y)} = e^{i e \Phi} \]

(\(\Phi\) is a "magnetic flux" penetrating the space \(S^1\) (A.-B. effect))

Thus, non-trivial topology (non-simply connected) of \(S^1\) plays an essential role to yield the finite \(m_H\)

(N.B.) In fact, in the case of \(S^2\), we have found a vanishing \(m_H\)
(toy model) 5D QED:

\[ A_M = (A_\mu, A_y) \ (A_y^{(0)} \propto H) \ , \ \text{electron} \quad \psi \ (m_\psi \equiv m) \]

(the cancellation of (quadratic) divergence of \( m_H^2 \) by K-K mode sum)

due to \(<A^{(0)}_y>\), electron gets “A-B phase” \( \alpha \):

\[ \psi(x^\mu, y + L) = e^{i \alpha} \psi(x^\mu, y) \quad (L = 2\pi R, \ \alpha = eL<A^{(0)}_y>) \]

\[ m_H^2 = \sum_{n=-\infty}^{\infty} \left( H \propto A_y^{(0)} \right) \psi(n) \sim H \]

\[ = ie^2 2^5 \sqrt{\frac{5}{2}} \int \frac{d^4k}{(2\pi)^4} \sum n \left\{ -\frac{1}{(\frac{2\pi n+\alpha}{L})^2 + \rho^2} + 2\rho^2 \frac{1}{[(\frac{2\pi n+\alpha}{L})^2 + \rho^2]^2} \right\} \]

\[ \left( \rho^2 \equiv -k^\mu \cdot k_\mu + m^2 \right) \]

using

\[ \frac{1}{L} \sum n \frac{1}{(\frac{2\pi n+\alpha}{L})^2 + \rho^2} = \frac{1}{2\rho} \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos \alpha} \]
\[ -ie^{2}\frac{2^{\frac{5}{2}}}{(2\pi)^{4}} \int \frac{d^{4}k}{(2\pi)^{4}} (1 + \rho \frac{\partial}{\partial \rho}) \{ (\frac{L}{2\rho}) \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos \alpha} \} \]

\[ = \frac{e^{2}}{3\pi^{2}} (\frac{1}{L^{2}}) \int_{0}^{\infty} ds \ s^{2} \frac{1 - \cosh \sqrt{s^{2} + (mL)^{2} \cdot \cos \alpha}}{[\cosh \sqrt{s^{2} + (mL)^{2} - \cos \alpha}]^{2}} \]

\[ (s \equiv \sqrt{-k^{\mu} \cdot k_{\mu} \cdot L}) \]

: super-convergent \[ \Rightarrow \text{finite } m_{H}^{2} \]

(N.B.) n=0 mode only \[ \Rightarrow \Lambda^{2} \text{- divergence} \]
(related topics)

- "dimensional deconstruction" : latticized 5D gauge theory
  @ $N \to \infty$ limit, the effective potential for $H$ coincides with what we obtained

- (ultra) natural inflation (N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, Phys.Rev.Lett. 90(’03)221302; T. Inami, Y. Koyama, S. Minakami &C.S.L., Progr. Theor. Phys. (09), to appear) : $A_y^{(0)}$ may be a natural candidate for the inflaton, as the local gauge symmetry stabilizes the potential under the quantum (gravity) correction

- little Higgs model : 4D theory, where $G/H$ of global symmetry provides Higgs as a N-G, may be “dual” to 5D GHU, where $A_y$ associated with $G/H$ of higher dimensional local gauge symmetry provides Higgs (“holographic principle”).
II. Gravity-Gauge-Higgs (GGH) unification scenario

(K. Hasegawa (Fumboldt Univ.), N. Maru (Chuo → Keio),

(our idea)

• the local symmetry, under which Higgs transforms inhomoneously, needs not to be gauge symmetry. It can be (higher dimensional) general coordinate invariance

• Gauge-Higgs unification → unification of s=1, s=0 bosons.

Is the unification of bosonic particles with all kinds of spins possible?

s=0    s=1    s=2

( H, A_μ, h_{μν} ) : Gravity-Gauge-Higgs unification
We attempt to realize the GGH unification in the framework of higher dimensional gravity theories, K-K type theories, in order to solve the hierarchy problem. The (zero-mode of) extra-space components of the metric tensor are identified with Higgs fields.

(a toy model)

- 5D K-K theory with a 5D massless scalar field $\Phi$ as a matter field

(purpose)

- we explicitly calculate the quantum correction to $m_H^2$ due to $\Phi$. (the quantum corrections due to particles with various spins, including the graviton itself, may be easily found just multiplying the physical degrees of freedom.)

- to confirm diagrammatically the cancellation mechanism of divergence in $m_H^2$

line element: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu - \exp(\Phi(dy + A_\mu dx^\mu))^2 \quad (x^\mu : 4D, \ y : \text{extra dim.})$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} : \ (4D)\text{graviton}$$

$$\phi = \phi_0 + h, \quad h : \ "\text{Higgs}" \quad (\phi_0 \equiv \langle \phi \rangle)$$

(N.B.) real Higgs is obtained by multiplying a constant to $h$, as $h$ is dimensionless
(specific sort of general coordinate transf.)

(a) \( y \rightarrow y + \alpha(x^\mu) \) leads to
\[ A_\mu \rightarrow A_\mu - \frac{\partial \alpha}{\partial x^\mu} \]
just as in QED: original idea of K-K

(b) \( y \rightarrow y + \alpha(y) \) leads to
\[ h \rightarrow h - 2\frac{\partial \alpha}{\partial y} \quad (A_\mu \rightarrow (1 + \frac{\partial \alpha}{\partial y})A_\mu) \]

Under (b), \( h \) transforms inhomogeneously, and \( \frac{1}{2}m_h^2h^2 \) term is forbidden

(metric)
\[
g_{MN} = \begin{pmatrix}
g_{\mu\nu} - e^\phi A_\mu A_\nu & -e^\phi A_\mu \\
-e^\phi A_\nu & -e^\phi
\end{pmatrix}
\]

\[
\frac{1}{2}m_h^2h^2 = \frac{1}{2\pi \hat{R}} \sum_n \left\{ \Phi_n + h + h - h \right\}
\]

where
\[
\hat{R} \equiv \text{Re} \frac{\phi}{2}, \quad \frac{1}{2\pi \hat{R}} = \frac{1}{2\pi \hat{R}_0}(1 - \frac{h}{2} + \frac{h^2}{8} - \ldots) \quad (\hat{R}_0 \equiv \text{Re} \frac{\phi_0}{2})
\]
\[ m_h^2 = \left(-\frac{i}{2}\right)\left(\frac{1}{2\pi \hat{R}_0}\right) \sum n \int \frac{d^4k}{(2\pi)^4} \times \left\{ \frac{1}{4} \ln \left[ -k_\mu k_\mu + \left(\frac{n}{\hat{R}_0}\right)^2 \right] - 2 \frac{(n/\hat{R}_0)^2}{k_\mu k_\mu - (n/\hat{R}_0)^2} - \frac{(n/\hat{R}_0)^4}{[k_\mu k_\mu - (n/\hat{R}_0)^2]^2} \right\} \]

denine function I obtained by K-K mode sum:

\[ I(\alpha, \hat{R}_0) \equiv \left(-\frac{i}{2}\right)\left(\frac{1}{2\pi \hat{R}_0}\right) \sum n \int \frac{d^4k}{(2\pi)^4} \ln \left[ -k_\mu k_\mu + \alpha^2 \left(\frac{n}{\hat{R}_0}\right)^2 \right] \]

\[ = \frac{1}{\alpha} [\widetilde{I} - \frac{3\alpha^5 \zeta(5)}{128\pi^7 \hat{R}_0^5}] \quad (\widetilde{I} : \text{divergent, for } \hat{R}_0 \to \infty) \]

thus,

\[ m_h^2 = \left[ \frac{1}{4} + \frac{1}{\alpha} \frac{d}{d\alpha} + \frac{1}{4} \left(\frac{1}{\alpha} \frac{d}{d\alpha}\right)^2 \right] I(\alpha, \hat{R}_0) \bigg|_{\alpha=1} \quad \left( \frac{d}{d\alpha^2} = \frac{1}{2\alpha} \frac{d}{d\alpha} \right) \]

\[ = \frac{-75 \zeta(5) 512\pi^7 \hat{R}_0^5}{512\pi^7 \hat{R}_0^5} \quad \text{divergent } \widetilde{I} \quad \text{term cancels out!} \quad \left( \zeta(5) = \sum_{n=1}^{\infty} \frac{1}{n^5} \right) \]

classical action for h tells us that physical Higgs H reads as

\[ H = \frac{\sqrt{6} h}{4 \kappa} \quad (\kappa = 8\pi G) \quad \rightarrow \quad m_H^2 = \frac{-25 \zeta(5)}{4\pi^5 M_{pl}^2 \hat{R}_0^4} \]