HIGHER SPIN ADS$_3$ GRAVITIES AND THEIR DUAL CFTS

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1. INTRODUCTION

Higher spin gauge theories and their applications to AdS/CFT correspondence

Higher spin gauge theories

- Higher spin gauge field
  - A totally symmetric spin-*s* field
  - Yang-Mills (*s*=1), gravity (*s*=2), …

$$\phi_{\mu_1\cdots\mu_s} \sim \phi_{\mu_1\cdots\mu_s} + \partial_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

- Vasiliev theory
  - Non-trivial interacting theories on AdS space
  - Only equations of motion are known
- Toy models of string theory in the tensionless limit
  - Singularity resolution
  - Simplified AdS/CFT correspondence

AdS/CFT correspondence

- Maldacena conjecture '97

Superstring theory on AdS_{d+1}	⟺	*d* dim. conformal field theory (CFT)

- Difficulties to proof the conjecture
  - Strong/week duality
  - Superstrings on AdS have not been solved
- Simplified AdS/CFT

Higher spin gauge theory on AdS_{d+1}	⟺	*d* dim. CFT with higher spin currents

Examples

- AdS_4/CFT_3 [Klebanov-Polyakov '02]

4d Vasiliev theory ⟺ 3d O(*N*) vector model

  - Evidences
    - Spectrum, RG-flow, correlation functions [Giombi-Yin '09, '10]
- AdS_3/CFT_2 [Gaberdiel-Gopakumar '10]

3d Vasiliev theory ⟺ Large *N* minimal model

  - Evidences
    - Symmetry, partition function, RG-flow, correlation functions
  - Supersymmetric extensions [Creutzig-YH-Rønne '11,'12]

Plan of the talk

1. Introduction
2. Higher spin gauge theories
3. Higher spin holography
4. Conclusion

2. HIGHER SPIN GAUGE THEORIES

Higher spin gravity theories and Chern-Simons formulation

Field equation (free theory)

- A totally symmetric spin-*s* field

$$\phi_{\mu_1\cdots\mu_s}$$

  - Yang-Mills (*s*=1), Gravity (*s*=2), …
- Field equations for free theory [Fronsdal '78]

$$\mathcal{F}_{\mu_1\ldots\mu_s} \equiv \Box\,\phi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1|}\partial^\lambda\phi_{|\mu_2\ldots\mu_s)\lambda} + \partial_{(\mu_1}\partial_{\mu_2}\phi_{\mu_3\ldots\mu_s)\lambda}{}^\lambda = 0$$

  - $\mathcal{F}_\mu = \partial^\nu F_{\nu\mu}$ (*s*=1), Linearized Ricci tensor (*s*=2)
- The higher spin gauge symmetry

$$\delta\phi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)},\ \ \xi^\lambda_{\lambda\mu_3\ldots\mu_s} = 0$$

  - Abelian gauge tfm. (*s*=1), Linearized diffeomorphism (*s*=2)

Action (free theory)

- The action for free theory

$$S = \frac{1}{2}\int d^D x \phi^{\mu_1\dots\mu_s}\left(\mathcal{F}_{\mu_1\dots\mu_s} - \frac{1}{2}\eta_{(\mu_1\mu_2}\mathcal{F}_{\mu_3\dots\mu_s)\lambda}{}^{\lambda}\right)$$

  - Uniquely fixed by the gauge symmetry
  - Under the double-traceless constraint

$$\phi^{\lambda\sigma}_{\lambda\sigma\mu_5\dots\mu_s} = 0$$

- Free theory on dS or AdS space [Fronsdal '79]

$$\partial_\mu \leftrightarrow \nabla_\mu, \;\; \mathcal{F}_{\mu_1\dots\mu_s} \leftrightarrow \hat{\mathcal{F}}_{\mu_1\dots\mu_s}$$

  - Derivatives are replaced by covariant derivative
  - The field strength receives corrections due to the curvature

Interacting theory

- Coleman-Mandula theorem
  - Any interacting theory is not possible with higher spin symmetry
  - Assumptions: mass gap, flat space, finitely many dof,…
- Vasiliev theory
  - Non-trivial interacting theory
    - Defined on AdS space
    - With all higher spins (s=2,3,4,…)
  - Only equations of motion are known
- Higher spin AdS_3 gravity
  - Topological theory, Chern-Simons description
  - Spin can be truncated (s=2,3,4,…N)

3d Einstein gravity

- Chern-Simons description [Achucarro-Townsend '86, Witten '88]
  - Actionof SL(2) x SL(2) CS theory

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$$

$$S_{\rm CS}[A] = \frac{k_{\rm CS}}{4\pi}\int {\rm tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right), \quad k_{\rm CS} = \frac{\ell}{4G}$$

  - Gauge transformation

$$\delta A = d\lambda + [A,\lambda], \ \ \delta\tilde{A} = d\tilde{\lambda} + [\tilde{A},\tilde{\lambda}]$$

$$A = A^a_\mu J_a dx^\mu, \ \ J_a (a=1,2,3) : {\rm sl}(2) \ \text{generator}$$

- Einstein Gravity with $\Lambda < 0$
  - Dreibein: $e^a_\mu = \frac{\ell}{2}(A^a_\mu - \tilde{A}^a_\mu), \ g_{\mu\nu} = \sum_{a=1}^3 e^a_\mu e^a_\nu$
  - Spin connection: $\omega_{\mu,a,b} = \frac{1}{2}\epsilon_{abc}\omega^c_\mu, \ \omega^c_\mu = \frac{1}{2}(A^c_\mu + \tilde{A}^c_\mu)$

Higher spin AdS_3 gravity

- G x G Chern-Simons theory
  - Higher spin gravity can be obtained by replacing SL(2) by G
- Embed gravitational sl(2) into g

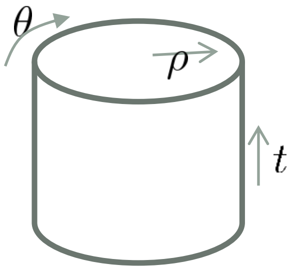
$$\mathrm{sl}(N) = \mathrm{sl}(2) \oplus \left(\bigoplus_{s=3}^{N} \mathrm{g}^{(s)} \right) \qquad (\text{c.f. } 8 = 3 + 5 \ \text{ for } \mathrm{SL}(3))$$

Gravitational sl(2) Space-time spin *s*

- Examples

Group G	Theory
SL(*N*)	Higher spin gravity with *s*=2,3,…,*N*
SL(∞)	BosonicVasiliev theory
SL(*N*+1|*N*)	Higher spin *N*=2 supergravity
SL(∞+1|∞)	SupersymmetricVasiliev theory [Blencowe '89]

Asymptotic symmetry



- Chern-Simons theory with boundary
  - DOF exist only at the boundary and described by WZNW model
- Classical asymptotic symmetry
  - Boundary conditions
    - Asymptotically AdS condition has to be assigned for AdS/CFT
    - The condition is equivalent to Drinfeld-Sokolov Hamiltonian reduction [Campoleoni, Fredenhagen, Pfenninger, Theisen '10, '11]
  - Examples

Group G	Symmetry	
SL(2)	Virasoro	Brown-Henneaux '86
SL(N)	W_N	Henneaux-Rey '10, Campoleni-Fredenhagen-Pfenninger-Theisen '10, Gaberdiel-Hartman '11
SL(N+1|N)	N=2 W_{N+1}	Creutzig-YH-Rønne '11, Henneaux-Gómez-Park-Rey '12, Hanaki-Peng '12

3. HIGHER SPIN HOLOGRAPHY

Proposals of simplified AdS/CFT correspondence and their evidences

AdS_4/CFT_3

- Klebanov-Polyakov conjecture '02

4d Vasiliev theory $\Longleftrightarrow$ 3d O(N) vector model

  - A weak/weak duality
- State counting

	Gauge invariant operator	Bulk fields
Vector-type model	$\sum_{a=1}^{N} h^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} h^a$	One higher spin field $\phi_{\mu_1 \cdots \mu_s}$
Matrix-type model	$\mathrm{tr}[\Phi \nabla^{l_1} \Phi \nabla^{l_2} \cdots \Phi]$	Many string states with fixed total spin

Evidences

- RG flow by a relevant operator $\mathcal{O} = \frac{\lambda}{2N}(h^a h^a)^2$

Flow	O(*N*) model	B.C. for bulk scalar
UV	Free theory ($\lambda = 0$)	Dirichlet (usual)
IR	Critical theory ($\lambda = \lambda^*$)	Neumann (alternative)

- Correlation functions
  - Some boundary correlation functions are computed explicitly from the bulk side [Giombi-Yin '09, '10]
  - A higher spin symmetry is enough to fix the CFT correlation functions [Maldacena-Zhiboedov '12, '12]

AdS_3/CFT_2

- Gaberdiel-Gopakumar conjecture ’10

3d Vasiliev theory	⟺	Large N minimal model

- Gravity side
  - A bosonic truncation of higher spin supergravity by Prokushkin-Vasiliev ’98
  - It includes massive scalar fields
- CFT side
  - Minimal model with respect to W_N-algebra (higher spin extension of Virasoro algebra)
  - Exactly solvable in principle

Minimal model holography

Higher spin gravity

- Massless sector
  - Higher spin gauge fields $(s = 2,3,\ldots)$
  - Asymptotic W_∞ symmetry
- Massive sector
  - Complex scalars

$$M^2 = -1 + \lambda^2$$

Large *N* minimal model

- W_N minimal model
  - Coset description

$$\frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$$

- 't Hooft limit
  - Large *N* limit

$$k, N \to \infty$$

  - Fix the ratio

$$0 < \lambda = \frac{N}{k+N} < 1$$

Evidences

- Symmetry
  - Asymptotic symmetry of the gravity theory is W algebra, while the dual CFT is W_N minimal model
- RG flow
  - RG flow pattern is reproduced from the bulk
- Spectrum
  - One loop partition functions of the dual theories match [Gaberdiel-Gopakumar-Hartman-Raju ’11]
- Interactions
  - Some three point functions are studied [Chang-Yin ’11, Ammon-Kraus-Perlmutter ’11]

$\mathcal{N}$=2 minimal model holography

- Our conjecture '12

> $\mathcal{N}$=2 Vasiliev theory ⟺ $\mathcal{N}$=(2,2) minimal model

- Gravity side
  - Full sector of higher spin supergravity by Prokushkin-Vasiliev '98
  - It includes fermionic higher spin gauge fields and massive matters addition to bosonic fields
- CFT side
  - Minimal model with respect to $\mathcal{N}$=(2,2) W_N-algebra
  - Given by $\mathcal{N}$=(2,2) CP^N Kazama-Suzuki model [Ito '91]

Our proposal

Prokushkin-Vasiliev theory

- Higher spin gauge fields
  - Bosons ($s = 1,2,\ldots$) and fermions ($s = 3/2, 5/2, \ldots$)
  - N=(2,2) W_∞ symmetry near the boundary of AdS_3
- Massivematter fields
  - Complex scalars
    $$(M_B^\pm)^2 = -1 + \frac{1}{4}(1 \mp 1 - 2\lambda)^2$$
  - Diracspin 1/2 spinors
    $$(M_F^\pm)^2 = (\frac{1}{2} - \lambda)^2$$

CP^N Kazama-Suzuki model

- N=(2,2) W_N minimal model
  - Coset description
    $$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)_{N(N+1)(k+N+1)}}$$
- 't Hooft limit
  - Large N limit
    $$k, N \to \infty$$
  - Fix the ratio
    $$0 < \lambda = \frac{N}{k+N} < 1$$

Evidences

- Symmetry
  - Asymptotic symmetry is $\mathcal{N}$=(2,2) $\mathcal{W}$ algebra [Creutzig-YH-Rønne ’11, Henneaux-Gómez-Park-Rey ’12, Hanaki-Peng ’12]
  - The Kazama-Suzuki model has the same symmetry [Ito ’91]
- Spectrum
  - One-loop partition function is obtained from gravity one-loop determinants [Creutzig-YH-Rønne ’11]
  - One loop partition function is computed at the ’t Hooft limit and the agreement is found with the gravity result [Candu-Gaberdiel ’12]
- Interactions
  - Three point functions with one higher spin current are studied [Creutzig-YH-Rønne, to appear]

Further generalizations

- SO(2*N*) holography [Ahn '11, Gaberdiel-Vollenweider '11]
  - Gravity side: Gauge fields with only spins *s*=2,4,6,…
  - CFT side: WD_{2N} minimal model

$$\frac{\mathrm{SO}(2N)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SO}(2N)_{k+1}}$$

- *N*=1 minimal model holography [Creutzig-YH-Rønne '12]
  - Gravity side: *N*=1 truncation of higher spin supergravity by Prokushkin-Vasiliev '98
  - CFT side: *N*=(1,1) S^{2N} model

$$\frac{\mathrm{SO}(2N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SO}(2N)_{k+1}}$$

$\mathcal{N}$=1 minimal model holography

Prokushkin-Vasiliev theory

- Higher spin gauge fields
  - Bosons ($s = 2,4, \dots$) and fermions ($s = 3/2, 5/2, \dots$)
  - $\mathcal{N}$=(1,1) W_∞ symmetry near the boundary of AdS_3
- Massivematter fields
  - Real scalars

$$(M_B^\pm)^2 = -1 + \tfrac{1}{4}(1 \mp 1 - 2\lambda)^2$$

  - Majoranaspin 1/2 spinors

$$(M_F^\pm)^2 = (\tfrac{1}{2} - \lambda)^2$$

$\mathcal{N}$=(1,1) S^{2N} model

- $\mathcal{N}$=(1,1) S^{2N} model
  - Coset description

$$\frac{\mathrm{SO}(2N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SO}(2N)_{k+1}}$$

- 't Hooft limit
  - Large N limit

$$k, N \to \infty$$

  - Fix the ratio

$$0 < \lambda = \frac{2N}{k+2N-1} < 1$$

Comments on $\mathcal{N}$=1 duality

- Spectrum
  - Gravity partition function can be reproduced by the 't Hooft limit of the dual CFT [Creutzig-YH-Rønne '12]
- Symmetry
  - $\mathcal{N}$=1 higher spin gravity
    - Gauge group is a large N limit of $OSP(2N+1|2N) \otimes OSP(2N+1|2N)$
    - Asymptotic symmetry from DS reduction
  - $\mathcal{N}$=(1,1) S^{2N} model
    - Generators of symmetry algebra are the same
    - (Anti-)commutation relations?

4. CONCLUSION

Summary and other related works

Summary

- Our conjecture
  - We propose a new AdS/CFT duality between higher spin $\mathcal{N}$=2 supergravity on AdS_3 by Prokushkin-Vasiliev and $\mathcal{N}$=(2,2) CP^N Kazama-suzuki model
- Strong evidences
  - Both theories have the same $\mathcal{N}$=(2,2) *W* symmetry
  - The partition function of gravity theory is reproduced by the 't Hooft limit of the dual CFT. This implies that the spectrum agrees.
- More evidences
  - Boundary 3-pt functions [Creutzig-YH-Rønne, in preparation]
  - $\mathcal{N}$=1 supersymmetric duality [Creutzig-YH-Rønne '12]
  - RG-flow
  - BPS sector

Other related works (I)

- Resolution of black hole singularity [Ammon-Gutperle-Kraus-Perlmutter '11,…]
  - The higher spin gravity has a large gauge symmetry. The notion of singularity, horizon,… is not gauge invariant
  - Generalized BTZ black hole can be changed into a warm hole solution by gauge transformation.
- 1/*N* corrections [Castro, Lepage-Jutier, Maloney '10,…]
  - Dual CFT is defined at the finite *N*, and the finite *N* effects should be related to the quantum effects of Vasiliev theory
  - Missing states in the CFT correspond to geometries with conical deficits

Other related works (II)

- 3d CS theory with vector matters [Aharony, Gur-Ari, Yacoby '11, Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin'11,…]
  - One parameter family of duality can be considered in the 't Hooft limit with large *N*,*k.*
  - A triality [Chang-Minwalla-Sharma-Yin '12, …]