Nonlinear massive gravity and Cosmology

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Based on collaboration with Antonio DeFelice, Emir Gumrukcuoglu, Chunshan Lin
Why alternative gravity theories?

- Afterglow Light Pattern 400,000 yrs.
- Dark Ages
- Development of Galaxies, Planets, etc.
- Inflation
- Dark Energy
- Big Bang
- "Singularity"
- Dark Matter

http://map.gsfc.nasa.gov/
Three conditions for good alternative theories of gravity
(my personal viewpoint)

1. Theoretically consistent
e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry
Some examples

I. Ghost condensation
   IR modification of gravity
   motivation: dark energy/matter

II. Nonlinear massive gravity
    IR modification of gravity
    motivation: “Can graviton have mass?”

III. Horava-Lifshitz gravity
     UV modification of gravity
     motivation: quantum gravity

IV. Superstring theory
    UV modification of gravity
    motivation: quantum gravity, unified theory
A motivation for IR modification

- Gravity at long distances
  - Flattening galaxy rotation curves
  - Extra gravity
  - Dimming supernovae
  - Accelerating universe

- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).
Dark component in the solar system?

Precession of perihelion observed in 1800’s…

which people tried to explain with a “dark planet”, Vulcan,

But the right answer wasn’t “dark planet”, it was “change gravity” from Newton to GR.
Can we change gravity in IR?

- **Change Theory?**
  - Massive gravity
    - Fierz-Pauli 1939
  - DGP model
    - Dvali-Gabadadze-Porrati 2000

- **Change State?**
  - Higgs phase of gravity
  - The simplest: Ghost condensation
Linear massive gravity (Fierz-Pauli 1939)

• Simple question: Can spin-2 field have mass?
  
• \[ L = L_{EH}[h] + m_g^2[\eta^{\mu\rho}\eta^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma}-(\eta^{\mu\nu}h_{\mu\nu})^2] \]

  \[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

• Unique linear theory without ghosts

• Broken diffeomorphism

\[ \rightarrow \text{no momentum constraint} \]

\[ \rightarrow \text{5 d.o.f. (2 tensor + 2 vector + 1 scalar)} \]
vDVZ vs Vainshtein

- van Dam-Veltman-Zhakharov (1970)
  Massless limit $\neq$ Massless theory = GR
  5 d.o.f remain $\rightarrow$ PPN parameter $\gamma = \frac{1}{2} \neq 1$
- Vainshtein (1972)
  Linear theory breaks down in the limit.
  Nonlinear analysis shows continuity and GR is recovered $@ r < r_\gamma=(r_g/m_g^4)^{1/5}$.
  Continuity is not uniform w.r.t. distance.
Naïve nonlinear theory and BD ghost

- FP theory with $\eta^{\mu\nu} \rightarrow g^{\mu\nu}$
  $$L = L_{EH}[h] + m_g^2 [g^{\mu\rho} g^{\nu\sigma} h_{\mu\nu} h_{\rho\sigma} - (g^{\mu\nu} h_{\mu\nu})^2]$$
  $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
- Vainshtein effect (1972)
- Boulware-Deser ghost (1972)
  No Hamiltonian constraint @ nonlinear level
  $$\rightarrow 6 \text{ d.o.f.} = 5 \text{ d.o.f. of massive spin-2} + 1 \text{ ghost}$$
Stuckelberg fields & Decoupling limit

• Stuckelberg scalar fields $\phi^a (a=0,1,2,3)$

$$g_{\mu\nu} = \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b + H_{\mu\nu} \quad \phi^a = x^a + \pi^a$$

$H_{\mu\nu}$: covariant version of $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

• Decoupling limit

$m_g \to 0, M_{pl} \to \infty$ with $\Lambda_5 = (m_g^4 M_{pl})^{1/5}$ fixed

• Helicity-0 part $\pi$: $\eta_{ab} \pi^b = \partial_a \pi$

sufficient for analysis of would-be BD ghost
Would-be BD ghost vs fine-tuning

Creminelli, Nicolis, Papucci & Trincherini 2005
de Rham, Gabadadze 2010

\[ H_{\mu \nu} = -2 \partial_\mu \partial_\nu \pi - \partial_\mu \partial_\rho \pi \partial_\rho \partial_\nu \pi \]

\[ h_{\mu \nu} = 0, \eta_{ab} \pi^b = \partial_a \pi \]

- **Fierz-Pauli theory**
  \( H_{\mu \nu}^2 - H^2 \)
  no ghost

- **3\text{rd} order**
  \( c_1 H_{\mu \nu}^3 + c_2 H H_{\mu \nu}^2 + c_3 H^3 \)
  no ghost if fine-tuned

- ... 

- any order
  no ghost if fine-tuned

Decoupling Helicity-0 limit part
Nonlinear massive gravity
de Rham, Gabadadze 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical
- Properties of 5 d.o.f. depend on background

- 4 scalar fields $\phi^a$ ($a=0,1,2,3$)
- Poincare symmetry in the field space:
  \[ \phi^a \rightarrow \phi^a + c^a, \quad \phi^a \rightarrow \Lambda_b^a \phi^b \]
  \[ f_{\mu \nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \]
  Pullback of Minkowski metric in field space to spacetime
Systematic resummation

de Rham, Gabadadze & Tolley 2010

\[ I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4 x \sqrt{-g} \left( \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 \right) \]

\[ f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \]

\[ \mathcal{K}_\mu^\nu = \delta_\mu^\nu - \left( \sqrt{g^{-1} f} \right)^\mu_\nu \]

\[ \mathcal{L}_2 = \frac{1}{2} \left( [\mathcal{K}]^2 - [\mathcal{K}^2] \right) \]

\[ \mathcal{L}_3 = \frac{1}{6} \left( [\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3] \right) \]

\[ \mathcal{L}_4 = \frac{1}{24} \left( [\mathcal{K}]^4 - 6 [\mathcal{K}]^2 [\mathcal{K}^2] + 3 [\mathcal{K}^2]^2 + 8 [\mathcal{K}] [\mathcal{K}^3] - 6 [\mathcal{K}^4] \right) \]

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

\[ \mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \rightarrow \mathcal{L}_{2,3,4} = (\text{total derivative}) \]

No BD ghost away from decoupling limit (Hassan&Rosen)
No FLRW universe?
D’Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

• Flat FLRW ansatz in “Unitary gauge”
  \[ g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2) \]
  \( \phi^a = x^a \quad \Rightarrow \quad f_{\mu\nu} = \eta_{\mu\nu} \)

• Bianchi “identity” \( \Rightarrow \) \( a(t) = \text{const.} \)
  c.f. \[ \nabla^\mu \left( \frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \right) = \frac{1}{\sqrt{-g}} \frac{\delta I_g}{\delta \phi^a} \partial_\nu \phi^a \]
  \( \Rightarrow \) no non-trivial flat FLRW cosmology

• “Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space”
Open FLRW solutions
Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\nu}$ spontaneously breaks diffeo.
- Both $g_{\mu\nu}$ and $f_{\mu\nu}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\nu}$
- No closed FLRW chart
- Open FLRW ansatz

\[
\begin{align*}
\phi^0 &= f(t)\sqrt{1 + |K|(x^2 + y^2 + z^2)}, \\
\phi^1 &= \sqrt{|K|} f(t)x, \\
\phi^2 &= \sqrt{|K|} f(t)y, \\
\phi^3 &= \sqrt{|K|} f(t)z.
\end{align*}
\]

\[
\begin{align*}
f_{\mu\nu}dx^\mu dx^\nu &= -\dot{f}(t)^2 dt^2 + |K| (f(t))^2 \Omega_{ij}(x^k)dx^i dx^j, \\
g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2 dt^2 + a(t)^2 \Omega_{ij} dx^i dx^j, \\
\Omega_{ij} dx^i dx^j &= dx^2 + dy^2 + dz^2 - \frac{|K|(xdx + ydy + zdz)^2}{1 + |K|(x^2 + y^2 + z^2)},
\end{align*}
\]
Open FLRW solutions
Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for $\phi^a$ (a=0,1,2,3)
  
  $$(\dot{a} - \sqrt{|K|}N) \left[ \left( 3 - \frac{2\sqrt{|K|}f}{a} \right) + \alpha_3 \left( 3 - \frac{\sqrt{|K|}f}{a} \right) \left( 1 - \frac{\sqrt{|K|}f}{a} \right) + \alpha_4 \left( 1 - \frac{\sqrt{|K|}f}{a} \right)^2 \right] = 0$$

- The first sol $\dot{a} = \sqrt{|K|}N$ implies $g_{\mu\nu}$ is Minkowski
  → we consider other solutions

  $$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

- Latter solutions do not exist if K=0

- Metric EOM → self-acceleration

  $$3 H^2 + \frac{3 K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2} \rho$$
  
  $$\Lambda_\pm \equiv - \frac{m_g^2}{(\alpha_3 + \alpha_4)^2} \left[ (1 + \alpha_3) (2 + \alpha_3 + 2 \alpha_3^2 - 3 \alpha_4) \pm 2 (1 + \alpha_3 + \alpha_3^2 - \alpha_4)^{3/2} \right]$$
Self-acceleration

\[ f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm = \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4} \]
General fiducial metric

- **Poincaré symmetry in the field space**
  \[ f_{\mu\nu} = (\text{Minkowski})_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \]

- **de Sitter symmetry in the field space**
  \[ f_{\mu\nu} = (\text{deSitter})_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \]

- **FRW symmetry in the field space**
  \[ f_{\mu\nu} = (\text{FLRW})_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \]

Flat/closed/open FLRW cosmology allowed if “fiducial metric” \( f_{\mu\nu} \) is de Sitter (or FRW)

- **Friedmann equation with the same effective cc**
  \[ 3 H^2 + \frac{3 K}{a^2} = \Lambda_{\pm} + \frac{1}{M_{Pl}^2} \rho \]

  \[ \Lambda_{\pm} \equiv -\frac{m_g^2}{(\alpha_3 + \alpha_4)^2} \left[ (1 + \alpha_3)(2 + \alpha_3 + 2 \alpha_3^2 - 3 \alpha_4) \pm 2(1 + \alpha_3 + \alpha_3^2 - \alpha_4)^{3/2} \right] \]
Cosmological perturbation with any matter


\[ I^{(2)} = \tilde{I}^{(2)}[Q_I, \Phi, \Psi, B_i, \gamma_{ij}] + \tilde{I}_{mass}^{(2)}[\psi^\pi, E^\pi, F^\pi_i, \gamma_{ij}] \]

\[ \tilde{I}[g_{\mu\nu}, \sigma_I] \equiv I_{EH,\tilde{\Lambda}}[g_{\mu\nu}] + I_{matter}[g_{\mu\nu}, \sigma_I] \quad \tilde{\Lambda} \equiv \Lambda + \Lambda_{\pm} \]

\[ \tilde{I}_{mass}^{(2)} = M_{Pl}^2 \int d^4x N a^3 \sqrt{\Omega} M_{GW}^2 \]

\[ M_{GW}^2 \equiv \pm(r - 1)m_g^2 X^2_{\pm} \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}, \]

\[ r \equiv \frac{\kappa a}{N \alpha} = \frac{1}{X_{\pm} H_f}, \quad H \equiv \frac{\dot{a}}{N a}, \quad H_f \equiv \frac{\dot{a}}{n \alpha} \]

\[ \times \left[ 3(\psi^\pi)^2 - \frac{1}{12} E^\pi \Delta(\Delta + 3K) E^\pi + \frac{1}{16} F^i_\pi(\Delta + 2K) F^\pi_i - \frac{1}{8} \gamma^{ij} \gamma_{ij} \right] \]

• **GR&matter part + graviton mass term**

• Separately gauge-invariant

Common ingredient is \( \gamma_{ij} \) only

• Integrate out \( \psi^\pi, E^\pi \) and \( F^\pi_i \) \( \rightarrow I^{(2)}_{s,v} = I^{(2)}_{GR s,v} \)

• Difference from GR is in the tensor sector only
Summary so far

• Nonlinear massive gravity
  free from BD ghost

• FLRW background
  No closed/flat universe
  Open universes with self-acceleration!

• More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$

• Cosmological linear perturbations
  Scalar/vector sectors $\rightarrow$ same as in GR
  Tensor sector $\rightarrow$ time-dependent mass
Nonlinear instability
DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc $\rightarrow$ FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation $\rightarrow$ nonlinear perturbation around flat FLRW

- **Odd-sector:**
  1 healthy mode + 1 healthy or ghosty mode

- **Even-sector:**
  2 healthy modes + 1 ghosty mode

- This is not BD ghost nor Higuchi ghost.
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New class of cosmological solution

• Healthy regions with (relatively) large anisotropy
• Are there attractors in healthy region?
• Classification of fixed points
• Local stability analysis
• Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.
→ Anisotropic FLRW universe!
statistical anisotropy expected
(suppressed by small $m_g^2$)
New class of cosmological solution

Anisotropy in Expansion

Anisotropy in fiducial metric
Summary

• Nonlinear massive gravity
  free from BD ghost
• FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
• More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
• Cosmological linear perturbations
  Scalar/vector sectors $\rightarrow$ same as in GR
  Tensor sector $\rightarrow$ time-dependent mass

• All homogeneous and isotropic FLRW solutions
  have nonlinear ghost
• New class of cosmological solution:
  anisotropic FLRW $\rightarrow$ statistical anisotropy
  (suppressed by small $m_g^2$)
• Analogue of Ghost Condensate!
Why alternative gravity theories?

1. Afterglow Light Pattern 400,000 yrs.
2. Dark Ages
3. Development of Galaxies, Planets, etc.
4. 1st Stars about 400 million yrs.
5. Big Bang Expansion 13.7 billion years
6. Big Bang "Singularity"
7. Inflation
8. Dark Energy
9. Dark Matter

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