Dynamical supersymmetry breaking from meta-stable vacua in an N=1 supersymmetric gauge theory

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Introduction

- Naturalness problem (fine-tuning problem)
  - The Standard Model (SM) is quite successful theory in particle physics.
  - Quarks, leptons, gauge bosons, Higgs
  - Non-zero Higgs VEV determines the vacuum.

\[ SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM} \]

W, Z bosons get masses of \( O(100) \) GeV
- Serious problem: SM vacuum is not stable under quantum corrections.

\[
\Delta m_H^2 = H \quad \quad H^+ \sim -\Lambda_{new}^2
\]

\[
\langle H \rangle \sim m_H^2 = \left( m_H^{tree} \right)^2 + \Delta m_H^2 \sim \left( m_H^{tree} \right)^2 - \Lambda_{new}^2 \quad \text{\( \Lambda_{new} \) is cut-off.} \]
Naturalness problem

- New physics scale >> electroweak scale

\[ \Lambda_{\text{new}} = M_{\text{Planck}} \sim 10^{19} \text{GeV} \]

\[ \langle H \rangle \sim (m_{H}^{\text{tree}})^2 - \Lambda_{\text{new}}^2 \]

\[ \Rightarrow O(M_W^2) = O((10^{19} \text{GeV})^2) - O((10^{19} \text{GeV})^2) \]

- What is a solution?
  - New physics without quadratic divergence in quantum corrections
    - Supersymmetry
Supersymmetry

- Supersymmetry (SUSY)
  - Symmetry that a boson and a fermion have the same mass spectra:
    - Ex: electron \(\rightarrow\) superpartner of electron
      (fermion) \(\rightarrow\) (sfermion)
  - Fine tuning problem can be solved.

\[ \Delta m_H^2 = f : \text{fermion} + \Lambda^2 \]

\[ \sim f : \text{sfermion} \]

\[ - \Lambda^2 \]
SUSY and unification

- SUSY favors unification of couplings
SUSY in future collider

- SUSY is expected to be observed at near future colliders
  - Large Hadron Collider (LHC)
  - International Linear Collider (ILC)

- A possible observation
  - Minimal SUSY SM
  - SUSY Grand Unified Theory
  - ...

However...
SUSY breaking

- **SUSY must be broken.**
  - No observation of superpartners of the SM particles yet (ex: superpartner of the electron)

- **How is SUSY broken?**
  - SUSY is broken by a scalar potential (**spontaneously breaking** as like gauge symmetry breaking by Higgs potential in the SM)
SUSY breaking and scalar potential

- An order parameter of SUSY breaking
  - SUSY ↔ Potential energy at vacuum is zero.
    \[ \langle 0 | \{ Q, \bar{Q} \} | 0 \rangle = \langle 0 | H | 0 \rangle = 0 \]
  - Breaking of SUSY ↔ Potential energy is non-zero.

SUSY vacuum

SUSY breaking vacuum
Spontaneous SUSY breaking

- How to realize SUSY breaking (not by hand but **spontaneously**)?
- O’Raifeartaigh model

\[
V(X, Y, Z) = 4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2
\]

**SUSY is spontaneously broken.**
Spontaneous SUSY breaking

- How to realize SUSY breaking (not by hand but spontaneously)?
- O’Raifeartaigh model

\[ V(X, Y, Z) = 4|SX + XZ|^2 + |X|^4 \]

Vacuum: \( X = 0 \)
Spontaneous SUSY breaking

- How to realize SUSY breaking (not by hand but spontaneously)?
- O’Raifeartaigh model

\[ V(X, Y, Z) = 4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2 \]

\[ V \]

Vacuum: \( X = 0 \)

Inconsistent with classical vacuum condition. SUSY is broken.

Origin?
Breaking by quantum corrections

- Dynamical symmetry breaking
  - Coleman-Weinberg mechanism (1972)

\[ V = \frac{\lambda}{4!} |\phi|^4 \]

\[ V = \frac{\lambda}{4!} |\phi|^4 + \frac{\lambda^2 \phi^2}{256\pi^2} \left( \ln \frac{\phi^2}{M^2} - \frac{25}{6} \right) \]

Tree \hspace{1cm} One loop level
Dynamical SUSY breaking

  - SUSY is not broken by perturbative corrections \(\rightarrow\) non-perturbatively

\[
\mathcal{L} = \int d^4 \theta K(X, X^\dagger) + \left( \int d^2 \theta W(X) + h.c. \right)
\]

Non-perturbative corrections can be derived by using holomorphy

\[
V(X, Y, Z) = 4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2
\]

Dynamical scale of gauge group
Dynamical SUSY breaking

  - SUSY is not broken by perturbative corrections $\rightarrow$ non-perturbatively

\[ \mathcal{L} = \int d^4 \theta K(X, X^\dagger) + \left( \int d^2 \theta W(X) + h.c. \right) \]

Non-perturbative corrections can be derived by using holomorphy

\[ V(X, Y, Z) = 4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2 \]

Any vacuum can be chosen.
Mass scale cannot be fixed.
E.g. Yukawa coupling: $\langle \Phi \rangle \bar{\psi} \psi = m \bar{\psi} \psi$
If $S + Z$ is the SM Higgs, it should be $O(100)$ GeV.

Vacuum is degenerate.
Dynamical SUSY breaking

  - SUSY is not broken by perturbative corrections \(\Rightarrow\) non-perturbatively

\[
\mathcal{L} = \int d^4 \theta K(X, X^\dagger) + \left( \int d^2 \theta W(X) + h.c. \right)
\]

Quantum corrections \(\downarrow\) Non-perturbative corrections can be derived by using holomorphy

\[
V(X, Y, Z) = f(X, Y, Z) \left( 4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2 \right)
\]

- Any vacuum can be chosen.
- Mass scale cannot be fixed.
  - E.g. Yukawa coupling: \(\langle \Phi \rangle \bar{\psi} \psi = m \bar{\psi} \psi\)
  - If \(S + Z\) is the SM Higgs, it should be \(O(100)\) GeV.
Dynamical SUSY breaking

  - SUSY is not broken by perturbative corrections → non-perturbatively

\[ \mathcal{L} = \int \! d^4 \theta K(X, X^\dagger) + \left( \int \! d^2 \theta W(X) + h.c. \right) \]

Quantum corrections

\[ V(X, Y, Z) = f(X, Y, Z)(4|SX + XZ|^2 + |X|^4 + |X^2 - \Lambda^2|^2) \]

Non-perturbative corrections can be derived by using holomorphy

• Corrections from Kähler potential can be estimated at best perturbatively in N=1 SUSY gauge theory.
• Such an estimation is possible only in weak coupling region.
  → Non-holomorphic

Weak coupling region

Strong coupling region

\( S + Z \)
N=2 SUSY

- N=1 SUSY gauge theory
  - Non-perturbative corrections to $W(X)$ can be calculable.
  - Effective $K(X, X^\dagger)$ is estimated at best by perturbative means.
N=2 SUSY

- N=1 SUSY gauge theory
  - Non-perturbative corrections to $W(X)$ can be calculable.
  - Effective $K(X, X^\dagger)$ is estimated at best by perturbative means.

- N=2 SUSY gauge theory
  - Possible to estimate non-perturbative corrections both for $K(X, X^\dagger)$ and $W(X)$
  - Seiberg, Witten 1994, 1995

- N=2 SUSY gauge theory perturbed by N=1 SUSY preserving mass term
  - Introducing mass term preserving only N=1 SUSY
    - N=2 SUSY is broken down N=1 SUSY
  - Assuming that the mass term does not affect to the results of SUSY.
    - Utilizing the exact result of N=2 SUSY
Purpose of our work

- Constructing N=2 SUSY model perturbed by N=1 preserving mass
  - SUSY is dynamically broken by gauge dynamics.
  - No pseudo flat direction.
Our model

- Lagrangian
  \[ \mathcal{L} = \mathcal{L}^{\mathcal{N}=2} + \mathcal{L}^{\mathcal{N}=1} \]
  \[ \mathcal{L}^{\mathcal{N}=2} : \text{N}=2 \text{ SUSY part} \]
  - SU(2)xU(1) gauge group with massless 2 flavors
    \[ \mathcal{L}^{\mathcal{N}=2} = \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{HM}} \]
    - \( \mathcal{L}_{\text{VM}} \) : gauge part
      - SU(2) vectormultiplet: \( V_2 \) (vector superfield), \( A_2 \) (chiral superfield)
        adjoint representation
      - U(1) vectormultiplet: \( V_1 \) (vector superfield), \( A_1 \) (chiral superfield)
    - \( \mathcal{L}_{\text{HM}} \) : matter part: 2 massless hypermultiplets
      - \( Q_r \) : chiral superfield ((2,1) representation under SU(2)xU(1)), \( r = 1, 2 \)
      - \( \tilde{Q}_r \) : chiral superfield ((2,-1) representation under SU(2)xU(1))
- \( \mathcal{L}^{\mathcal{N}=1} : \text{N}=1 \text{ SUSY preserving part} \)
  - Mass term for \( A_2 \) and \( A_1 \)
  - Linear term in \( A_1 \) (Fayet-Iliopoulos term, not breaking SUSY)
Our model

- **Lagrangian** \( \mathcal{L} = \mathcal{L}^{N=2} + \mathcal{L}^{N=1} \)
  
  \( \mathcal{L}^{N=2} = \mathcal{L}_{VM} + \mathcal{L}_{HM} \)
  
  \( \mathcal{L}_{VM} = \frac{1}{2\pi} \text{Im} \left[ \text{Tr} \left\{ \tau_{22} \left( \int d^4\theta \ A_2^\dagger e^{2V_2} A_2 e^{-2V_2} + \frac{1}{2} \int d^2\theta \ W_2^2 \right) \right\} \right] \)
  
  \( + \frac{1}{4\pi} \text{Im} \left[ \tau_{11} \left( \int d^4\theta \ A_1^\dagger A_1 + \frac{1}{2} \int d^2\theta \ W_1^2 \right) \right] \)

  Coupling constants: \( \tau_{22} = i \frac{4\pi}{g^2} + \frac{\theta}{2\pi} \), \( \tau_{11} = i \frac{4\pi}{e^2} \)

  \( \mathcal{L}_{HM} = \int d^4\theta \left( Q_r^\dagger e^{2V_2+2V_1} Q^r + \tilde{Q}_r e^{-2V_2-2V_1} \tilde{Q}^r \right) \)
  
  \( + \sqrt{2} \left( \int d^2\theta \ \tilde{Q}_r (A_2 + A_1) Q^r + \text{h.c.} \right) \).

- \( \mathcal{L}^{N=1} = \int d^2\theta \left( \mu_2 \text{Tr}(A_2^2) + \frac{1}{2} \mu_1 A_1^2 + \lambda A_1 \right) + \text{h.c.} \)

  **Fayet-Iliopoulos term**
Potential analysis

- **Scalar potential**
  
  \[ V = V(A_2, A_1, Q, \tilde{Q}, \mu_1, \mu_2, \lambda) \]

  - Purely N=2 case \((\mu_i = \lambda = 0)\)

  \[ Q = \tilde{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1 \]

  - \(SU(2) \times U(1) \Rightarrow U(1)_{c} \times U(1)\) (Coulomb branch)

  - Turning on N=1 preserving mass term \((\mu_i \neq 0, \lambda \neq 0)\)

  \[ Q_r = \tilde{Q}_r = 0, \quad A_2 = 0, \quad A_1 = -\frac{\lambda}{\mu_1} \]

  - \(SU(2)\) gauge symmetry recovers.

  - Vacuum on Higgs branch

  \[ Q_r \neq 0, \quad \tilde{Q}_r \neq 0, \quad A_2 = 0, \quad A_1 = 0 \]
Potential analysis

- Scalar potential
  \[ V = V(A_2, A_1, Q, \tilde{Q}, \mu_1, \mu_2, \lambda) \]
  - Purely N=2 case \((\mu_i = \lambda = 0)\)
  \[ Q = \tilde{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1 \]
  - \(SU(2) \times U(1) \Rightarrow U(1)_c \times U(1)\) (Coulomb branch)
  - Turning on N=1 preserving mass term \((\mu_i \neq 0, \lambda \neq 0)\)
  \[ Q_r = \tilde{Q}_r = 0, \quad A_2 = 0, \quad A_1 = -\frac{\lambda}{\mu_1} \]
  - \(SU(2)\) gauge symmetry recovers.
  - Vacuum on Higgs branch
    \[ Q_r \neq 0, \quad \tilde{Q}_r \neq 0, \quad A_2 = 0, \quad A_1 = 0 \]

With quantum corrections
Quantum theory

- Low energy Wilsonian effective action
  - Integrating out heavy fields
    \[ S = \int_{|k| > \Lambda} \Pi_i \mathcal{D}\phi_i e^{i \int \mathcal{L}(\phi_i)} \]
  - Effective action is described by light fields, \( \mu_i, \lambda \), dynamical scale of SU(2) gauge group \( \Lambda \) (introduced by dimensional transmutation)
    \[ \mathcal{L}_{\text{exact}} = \mathcal{L}(\phi_i, \mu_i, \lambda, \Lambda) \]
    Difficult task
  - Assuming that \( \mu_i \ll \Lambda, \lambda \ll \Lambda \)
    \[ \mathcal{L}_{\text{exact}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{mass}} + \mathcal{O}(\mu_i, \lambda) \]
    Leading order in \( \mu_i, \lambda \)

N=2 SUSY part \( (\mu_i = \lambda = 0) \)
Quantum theory

- Low energy effective action – N=2 SUSY part

\[ \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{VM} + \mathcal{L}_{HM} \]

- General form respecting \( U_c(1) \times U(1) \) gauge symmetry

\[
\mathcal{L}_{VM} = \frac{1}{4\pi} \text{Im} \sum_{i,j=1}^{2} \left[ \int d^4 \theta \frac{\partial \mathcal{F}}{\partial A_i} A_i^\dagger + \frac{1}{2} \int d^2 \theta \ \tau_{ij} W_i W_j \right]
\]

Prepotential: \( \mathcal{F} = \mathcal{F}(A_1, A_2, \Lambda, \Lambda_L) \quad A_2 \in U(1)_C \)

\( \longleftrightarrow \) determined by vacuum information (discussed below)

\( \longleftrightarrow \) described by elliptic function

Coupling constants: \( \tau_{ij} = \frac{\partial^2 \mathcal{F}}{\partial a_i \partial a_j} \quad (i,j = 1,2) \)
Quantum theory

- **N=2 SUSY part** \( \mathcal{L}_{\text{SUSY}} \)
  
  - Deriving low energy action at each point of vacuum \( \rightarrow \) understanding of moduli

\[
Q = \bar{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1
\]
Quantum theory

- **N=2 SUSY part** $\mathcal{L}_{\text{SUSY}}$
  - Deriving low energy action at each point of vacuum understanding of moduli

\[ \mathcal{L}(a_2^l(u), a_1^l, q, \bar{q}) \]

\[ Q = \bar{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1 \]

\[ \mathcal{L}(a_2(u), a_1, q, \bar{q}) \]

\[ u \to \text{Tr}((A_2)^2) \]
Quantum theory

- N=2 SUSY part $\mathcal{L}_{\text{SUSY}}$
  - Deriving low energy action at each point of vacuum $\Rightarrow$ understanding of moduli

$$\mathcal{L}(a_2'(u), a_1', q, \tilde{q})$$

$$Q = \tilde{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1$$

$$\mathcal{L}(a_2(u), a_1, q, \tilde{q})$$

$g \ll 1$

Weak coupling region
(perturbative means works well)
Quantum theory

- N=2 SUSY part $\mathcal{L}_{SUSY}$
  - Deriving low energy action at each point of vacuum $\Rightarrow$ understanding of moduli

\[
\mathcal{L}(a^I_2(u), a^I_1, q, \bar{q})
\]

Singular point: $g(\Lambda) \to \infty \not\in$ Strong coupling region

$Q = \bar{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1$

Weak coupling region (perturbative means works well)

$\mathcal{L}(a^I_2(u), a^I_1, q, \bar{q})$

$g \gg 1$

$g \ll 1$
Quantum theory

- **N=2 SUSY part** $\mathcal{L}_{\text{SUSY}}$
  - Deriving low energy action at each point of vacuum understanding of moduli

  $\mathcal{L}(a_2'(u), a_1', q, \bar{q})$

  **Singular point:** $g(\Lambda) \to \infty$

  **Strong coupling region**
  
  $g' \equiv 1/g$

  $\frac{1}{g'^2} \int d^4 x F_{D}^{\mu \nu} F_{D \mu \nu}$

  **Weak coupling region**
  
  $g \ll 1$

  $\frac{1}{g^2} \int d^4 x F^{\mu \nu} F_{\mu \nu}$

  **Duality**

  $Q = \bar{Q} = 0$, $A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}$, $A_1 = a_1$
Quantum theory

- N=2 SUSY part $\mathcal{L}_{\text{SUSY}}$
  - Deriving low energy action at each point of vacuum $\Rightarrow$ understanding of moduli

\[ \mathcal{L}(a_{2D}(u), a_{1D}(u), q, \bar{q}) \]

Singular point: $g(\Lambda) \to \infty \times$

Strong coupling region

\[ \frac{1}{g^2} \int d^4x F_D^{\mu \nu} F_D_{\mu \nu} \]

Dual variable

\[ a_{2D} = \frac{\partial F}{\partial a_2}, \quad a_{1D} = \frac{\partial F}{\partial a_1} \]

Weak coupling region (perturbative means works well)

\[ \frac{1}{g^2} \int d^4x F_{\mu \nu} F_{\mu \nu} \]

Action is invariant under the trans of the set of variables $\Rightarrow$ monodromy transformation

\[ (a_{2D}(u), a_{1D}(u), a_2(u), a_1) \]

$\rightarrow M(a_{2D}(u), a_{1D}(u), a_2(u), a_1)$
Quantum theory

- N=2 SUSY part \( \mathcal{L}_{\text{SUSY}} \)
  - Deriving low energy action at each point of vacuum understanding of moduli

\[
\mathcal{L}(a_{2D}(u), a_{1D}(u), q, \tilde{q})
\]

\[
Q = \tilde{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1
\]

\[
\mathcal{L}(a_2(u), a_1, q, \tilde{q})
\]

Singular point: \( g(\Lambda) \to \infty \)

- Strong coupling region
  - \( g \gg 1 \)
  - Landau pole
  - \( \Lambda_L \)
  - Dual variable
  - \( a_{2D} = \frac{\partial F}{\partial a_2}, \quad a_{1D} = \frac{\partial F}{\partial a_1} \)

- Weak coupling region (perturbative means works well)
  - \( g \ll 1 \)
  - Weak coupling region
  - Action is invariant under the trans of the set of variables monodromy transformation

\[
\frac{1}{g^2} \int d^4 x F_D^{\mu \nu} F_{D \mu \nu}
\]

\[
\frac{1}{g^2} \int d^4 x F^{\mu \nu} F_{\mu \nu}
\]

- U(1) gauge coupling is always weak in the region \( a_1 < \Lambda_L \).

- U(1) gauge dynamics does not affect SU(2) gauge dynamics
Quantum theory

- Low energy effective action – N=2 SUSY part

\[ \mathcal{L}_{\text{SU2}} = \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{HM}} \]

- General form respecting \( U_c(1) \times U(1) \) gauge symmetry

\[ \mathcal{L}_{\text{VM}} = \frac{1}{4\pi} \text{Im} \sum_{i,j=1}^{2} \left[ \int d^4\theta \frac{\partial \mathcal{F}}{\partial A_i} A_i^\dagger + \frac{1}{2} \int d^2\theta \tau_{ij} W_i W_j \right] \]

**Prepotential:** \( \mathcal{F} = \mathcal{F}(A_1, A_2, \Lambda, \Lambda_L) \)

\[
\mathcal{F} = \mathcal{F}_{\text{SQCD}}^{SU(2)}(A_2, m, \Lambda) \bigg|_{m=\sqrt{2}A_1} + c A_1^2
\]

N=2 SU(2) SQCD effective action \( c \) includes the info of Landau pole.

**Dual variable:** \( A_{2D} = \frac{\partial \mathcal{F}}{\partial A_2}, \quad A_{1D} = \frac{\partial \mathcal{F}}{\partial A_1} \)

**Coupling constants:** \( \tau_{ij} = \frac{\partial^2 \mathcal{F}}{\partial a_i \partial a_j} \quad (i, j = 1, 2) \)
Quantum theory

- Low energy effective action – N=2 SUSY part

\[ \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{HM}} \]
Quantum theory

- N=2 SUSY part $\mathcal{L}_{\text{SUSY}}$
  - Deriving low energy action at each point of vacuum $\Rightarrow$ understanding of moduli

\[ \mathcal{L}(a_{2D}(u), a_{1D}(u), M, \tilde{M}) \]

$q, \tilde{q}$ are no longer light fields around a singular point. A new light fields $M, \tilde{M}$ appear. (Monopole, dyon)

Weak coupling region (perturbative means works well)

\[ Q = \tilde{Q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1 \]

\[ \mathcal{L}(a_2(u), a_1, q, \tilde{q}) \]

Strong coupling region

\[ \frac{1}{g^2} \int d^4 x F_{D}^{\mu \nu} F_{D \mu \nu} \]

\[ g'(\equiv 1/g) \]

U(1) gauge coupling is always weak in the region $a_1 < \Lambda_L$. U(1) gauge dynamics does not affect SU(2) gauge dynamics

\[ (a_{2D}(u), a_{1D}(u), a_2(u), a_1) \rightarrow M(a_{2D}(u), a_{1D}(u), a_2(u), a_1) \]

Action is invariant under the trans of the set of variables $\Rightarrow$ monodromy transformation
Quantum theory

- Low energy effective action – N=2 SUSY part

\[ \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{HM}} \]

- Hypermultiplet part

\[ \mathcal{L}_{\text{HM}} = \int d^4 \theta \left[ M^r e^{2n_m V_2 + 2n_e V_2 + 2n_1 V_1 M^r} + \tilde{M}^r e^{-2n_m V_2 - 2n_e V_2 - 2n_1 V_1 \tilde{M}^r} \right] \]

\[ + \sqrt{2} \int d^2 \theta \left[ \tilde{M} (n_m A_{2D} + n_e A_2 + n A_1) M^r + h.c. \right] \]

\( M, \tilde{M} \) : Quark, monopole, dyon

\( (n_m, n_e)_n \) : Magnetic, electric charges, U(1) charge.

Ex. Quark: \( (n_m, n_e)_n = (0, 1)_1 \)
Quantum theory

- N=1 SUSY preserving part

\[ \mathcal{L}_{\text{mass}} = \int d^2 \theta \left[ \mu_2 U(A_1, A_2) + \frac{1}{2} \mu_1 A_1^2 + \lambda A_1 \right] + h.c. \]

\[ U(A_2, A_1) = u(a_2, a_1) + \theta^2 F_2 \frac{\partial u}{\partial a_2} \bigg|_{a_1} + \theta^2 F_1 \frac{\partial u}{\partial a_1} \bigg|_{a_2} \]
Quantum theory

\begin{itemize}
\item N=1 SUSY preserving part
\end{itemize}

\[ \mathcal{L}_{\text{mass}} = \int d^2 \theta \left[ \mu_2 U(A_1, A_2) + \frac{1}{2} \mu_1 A_1^2 + \lambda A_1 \right] + h.c. \]

\[ U(A_2, A_1) = u(a_2, a_1) + \text{(fermion, auxiliary fields)} \]
Potential analysis

- Effective scalar potential

\[ V = V(a_2(u), a_1, M, \tilde{M}) \]

- In the following, we assume that the potential is described by the proper variables associated with the light states.
- Solving stationary condition with respect to \( M, \tilde{M} \)

\[ 0 = \frac{\partial V}{\partial M} = \frac{\partial V}{\partial \tilde{M}} \]

\[
\begin{align*}
1) \quad & V(a_2(u), a_1) = Y(a_2, a_1), \\
2) \quad & V(a_2(u), a_1) = Y(a_2, a_1) - 4S(a_2, a_1)M(a_2, a_1)^4 \\
& S(a_2, a_1) > 0 \quad M \equiv M = \tilde{M}
\end{align*}
\]

- Potential minimum is energetically favored if light matter acquires VEV.
- Light matter appears only around a singular point.
Singular points

- In our model, there are 3 singular points associated with light fields.

\[ u_1 = -\sqrt{2a_1} \Lambda - \frac{\Lambda^2}{8} \]

\[ u_2 = \sqrt{2a_1} \Lambda - \frac{\Lambda^2}{8} \]

\[ u_3 = 4a_1^2 + \frac{\Lambda^2}{8} \]
Singular points and vacua

- In our model, there are 3 singular points associated with light fields.

\[ u_1 = -\sqrt{2}a_1\Lambda - \frac{\Lambda^2}{8} \]

\[ u_2 = \sqrt{2}a_1\Lambda - \frac{\Lambda^2}{8} \]

\[ u_3 = 4a_1^2 + \frac{\Lambda^2}{8} \]
Singular points and vacua

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Singular points and vacua

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Singular points and vacua

- In our model, there are 3 singular points associated with light fields.
Why is SUSY broken?

- Slice along $a_1$ direction

\[ a_1 \sim -\frac{\lambda}{\mu_1} \]
Why is SUSY broken?

- Slice along $a_1$ direction

\begin{align*}
V & \\
\frac{a_1}{a_1} & \sim -\frac{\lambda}{\mu_1} \\
\end{align*}

Tree level
\[ \mu_i \neq 0, \quad \lambda \neq 0 \]

\[ a_1 = -\frac{\lambda}{\mu_1} \]

Quantum level
\[ \mu_2 \neq 0, \quad \mu_1 = \lambda = 0 \]
Decay rate

- **Decay to Higgs branch**
  - SUSY vacuum in Higgs branch
  - Our SUSY breaking vacuum is local.
  - Reasonable parameter region where the local vacuum at Coulomb branch is meta-stable.

- **SUSY breaking vacuum (Coulomb branch)**

\[(a_2, a_1) = \left( \Lambda, \frac{\lambda}{\mu_1} \right)\]

- **SUSY vacuum (Higgs branch)**

Intriligator-Seiberg-Shih, 2006

- **Parameter choice**

\[
\begin{align*}
\mu_i & \ll \Lambda, \quad \lambda \ll \Lambda \\
\Lambda & \gg \lambda/\mu_1, \quad \lambda/\mu_2^2 \gg g^2
\end{align*}
\]

- Decay rate is extremely small.

\[
\Gamma \sim e^{-B}, \quad B \gg 1
\]
Conclusion

- We investigated N=2 SU(2)xU(1) gauge theory with 2 massless flavors with N=1 SUSY preserving mass terms and a Fayet-Iliopoulos term.

- We derived the exact low energy effective action by utilizing non-perturbative analysis based on Seiberg-Witten solution.

- We found that SUSY is dynamically broken on the Coulomb branch, where no pseudo flat direction exists.

- There is also a SUSY vacuum, so this is a local vacuum.

- We found that the local vacuum is meta-stable in a certain parameter choice in the theory.