The Fall of the Black Hole Firewall:
Quantum Information of Black Holes

**Based on**

M. Hotta and A. Sugita, arXiv:1505.05870

M. Hotta, R. Schützhold and W. G. Unruh,

M. Hotta, J. Matsumoto and K. Funo,

Part I
INFORMATION LOSS PROBLEM
No Firewall in Typical States
Inevitable Modification of Page Curve
Zero-Point Fluctuation as Partner of Hawking Particle

Part II
ZERO-POINT FLUCTUATION AND BLACK HOLES
Quantum Energy Teleportation and Black Hole Entropy
Quantum Informational Cosmic Censorship Conjecture

If my time is up, I will suddenly stop my talk, sorry...
Part I

INFORMATION LOSS PROBLEM
The Information Loss Problem

Hawking (1976)

\[ \hat{U} |\Psi\rangle \langle \Psi| \hat{U}^\dagger \neq \rho_{\text{thermal}} \]

Unitarity breaking?

Information is lost!?
Why is the information loss problem so serious?

Too small energy to leak the huge amount of information.
(Aharonov, et al 1987; Preskill 1992.)

If the horizon prevents enormous amount of information from leaking until the last burst of BH, only very small amount of BH energy remains, which is not expected to excite carriers of the information and spread it out over the outer space.
Purification Problem of Hawking Radiation: from a modern viewpoint of quantum information

\[ \rho_{HR} = \sum_{n} p_n \ket{n}_{HR} \bra{n}_{HR} \]

Mixed state

Composite system in a pure state

\[ \ket{\Psi}_{HRA_{HR}} = \sum_{n} \sqrt{p_n} \ket{n}_{HR} \ket{u_n}_{A_{HR}} \]

Composite system in a pure state
What is the final purification partner of the Hawking radiation?

(1) Nothing, Information Loss

(2) Exotic Remnant (Aharonov, Banks, Giddings,...)

(3) Baby Universe (Dyson,..)

(4) Radiation Itself (Page,...)
   ○ Black Hole Complementarity (‘t Hooft, Susskind, ...)
   ○ Fuzzi ball, Firewall (Mathur, Braunstein, AMPS, ...)


Black Hole Complementarity

From the viewpoint of free-fall observers, no drama happens across the horizon.
Black Hole Complementarity

From the viewpoint of outside observers, the stretched horizon absorbs and emits quantum information so as to maintain the unitarity.
A **FIREWALL** on the horizon burns out free-fall observers. The inside region of BH does not exist! This is argued from monogamy of entanglement.

This scenario is criticized in this talk.
What is the final purification partner of the Hawking radiation?

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(4) Radiation Itself (Page,..)
   ○ Black Hole Complementarity (‘t Hooft, Susskind, ...)
   ○ Fuzzi ball, Firewall (Mathur, Braunstein, AMPS, ...)

(5) Zero-Point Fluctuation Flow
   (Wilczek, Hotta-Schützhold-Unruh, Hawking (2015) )

Gravitational zero-point fluctuation with BMS charges
Canonical typicality for non-vanishing Hamiltonians yields non-maximal entanglement among black holes and the Hawking radiation, which makes spacetimes smooth without breaking monogamy. Thus, no reason to have BH firewalls.
Typical states must be Gibbs states for smaller quantum systems with very high precision. If we have stable Gibbs states for old evaporating BH’s, the heat capacity must be positive. Actually, it is negative. Thus the states of BH evaporation are not typical!

Inevitable Modification of the Page Curve
Microcanonical states $\rho \propto I_E$ are far from typical for finite-temperature old BH’s, even though canonical states are typical and the large entropy $O(V)$ is merely different from the microcanonical entropy by $O(\ln V)$. Entropy difference between a typical state and the canonical state must be exponentially small!

\[ |S_{\text{typical}} - S_{\text{thermal}}| \leq O(\exp(-\gamma V)) \sim 0 \]
You cannot use the microcanonical state in the typicality argument for evaporating black holes.

\[
\rho_{BH} \propto \frac{E}{E - \delta}
\]
Plan of Part I

1. Lubkin-Lloyd-Pagels-Page Theorem, Page Curve Hypothesis and BH Firewall Conjecture

2. Canonical Typicality for Non-Vanishing Hamiltonians Yields No Firewalls

3. Summary of Part I
1. Lubkin-Lloyd-Pagels-Page Theorem, Page Curve Hypothesis and BH Firewall Conjecture
**Entanglement Entropy** as a measure of entanglement between two systems in a **pure state**:

\[ N_A \quad \Psi_{AB}^A \quad N_B \]

**Reduced States:**

\[ \rho_A = \text{Tr}_B \left[ \Psi_{AB}^A \Psi_{AB}^A \right], \quad \rho_B = \text{Tr}_A \left[ \Psi_{AB}^A \Psi_{AB}^A \right] \]

\[ S_{EE} \left( \Psi_{AB}^A \Psi_{AB}^A \right) = - \text{Tr}_A \left[ \rho_A \ln \rho_A \right] = - \text{Tr}_B \left[ \rho_B \ln \rho_B \right] \]
Schmidt Decomposition of Pure State:

\[ |\Psi_{AB}\rangle = \sum_{n} \sqrt{p_{\Psi_n}} |u_n\rangle_A |v_n\rangle_B \]

\[ p_{\Psi_n} \geq 0, \sum_{n} p_{\Psi_n} = 1, \langle u_n | u_{n''} \rangle = \delta_{nn'}, \langle v_n | v_{n'} \rangle = \delta_{nn'} \]

\[ \Rightarrow S_{EE} \left( \left| \Psi_{AB} \right\rangle \langle \Psi_{AB} \right| \right) = - \sum_{n} p_{\Psi_n} \ln p_{\Psi_n} \]

Example: The Bell states attain the maximum value of entanglement entropy.

\[ |Bell\rangle = \frac{1}{\sqrt{2}} \left[ |+\rangle + |\rangle + |\rangle - |\rangle - \right] \]

\[ \Rightarrow S_{EE} \left( |Bell\rangle \langle Bell| \right) = \ln 2 \]
In the case with $N_A \leq N_B$, the maximum value of $EE$ is

$$S_{EE} = \ln N_A$$
The **BH firewall** conjecture is based on the **Page curve** hypothesis, and the hypothesis was inspired by the **Lubkin-Lloyd-Pagels-Page (LLPP)** theorem for entanglement entropy.
**Lubkin-Lloyd-Pagels-Page Theorem:**

Typical states of A and B are almost maximally entangled when the systems are large.

Typical State of AB

\[ \rho_A = Tr_B \left[ |\Psi\rangle_{AB} \langle \Psi_{AB}| \right] \]

\[ \langle S_{EE} \rangle = -\langle Tr_A [\rho_A \ln \rho_A] \rangle \]

Almost Max Value of EE

\[ \langle S_{EE} \rangle \approx \ln N_A \]

\[ \rho_A \approx \frac{1}{N_A} I_A \]
Maximal Entanglement between $A$ and $B$

$$\rho_A = \frac{1}{N_A} I_A \Rightarrow\quad |\text{Max}\rangle_{AB} = \frac{1}{\sqrt{N_A}} \sum_{n=1}^{N_A} |u_n\rangle_A |\tilde{v}_n\rangle_B$$

Orthogonal unit vectors

$$N_B \geq N_A$$
Let us assume that Hilbert-space dimensions of black holes and Hawking radiation become \textit{finite} due to quantum gravity effect.

\textbf{Page’s Strategy for Finding States of BH Evaporation:} Nobody knows exact quantum gravity dynamics. So let’s gamble that the state scrambled by quantum gravity is one of \textit{TYPICAL} pure states of the finite-dimensional composite system! That may not be so bad!
Proposition I:
When the dimension of the BH Hilbert space is much larger or less than that of Hawking radiation, BH and HR in a typical pure state of quantum gravity share almost maximal entanglement. In other words, quantum states of the smaller system is almost proportional to the unit matrix.

Proposition II:
\[ S_{EE} = S_{\text{thermal}} \] of the smaller system.
\[ |BH| = \text{dim } H_{BH}, \] 
\[ |HR| = \text{dim } H_{HR}, \]

<<Page Time>>

\[ \ln |BH| \approx \ln |HR| \]

\[ M_{\text{page}} \approx 0.7 M_{bh} \]

\[ 1 \ll |BH| \ll |HR| \Rightarrow \langle S_{EE} \rangle \approx \ln |BH| = \frac{A_{BH}}{4G} \]
\( H R = A \cup B \)
\( BH = C \)

\( 1 << |A|, |B|, |C| \)

\( OLD BH \Rightarrow |B| |C| << |A| \)
Proposition I means that $A$ and $BC$ are almost maximally entangled with each other.

$$\rho_{BC} = \frac{1}{|BC|} I_{BC} = \left( \frac{1}{|B|} I_B \right) \otimes \left( \frac{1}{|C|} I_C \right)$$

NO CORRELATION BETWEEN B AND C!
\[ \rho_{BC} = \left( \frac{1}{|B|} I_B \right) \otimes \left( \frac{1}{|C|} I_C \right) \]

\[
\text{Tr}\left[ \left( \frac{\varphi(x_B) - \varphi(x_C)}{\varepsilon} \right)^2 \rho_{BC} \right] = O\left( \frac{1}{\varepsilon^2} \right)
\]

\[ \varepsilon \to 0 \]

\[ \text{Tr}\left[ (\partial \varphi(x))^2 \rho_{BC} \right] = \infty \]

\text{FIREWALL!}
Strong Sub-additivity (Monogamy) Paradox
**Typical-State Condition**

\[
\rho_{BC} = \left( \frac{1}{|B|} I_B \right) \otimes \left( \frac{1}{|C|} I_C \right)
\]

\[
\rho_B = \frac{1}{|B|} I_B \implies \left| \text{Max} \right\rangle_{AB} = \frac{1}{\sqrt{|B|}} \sum_{n=1}^{|B|} \left| u_n \right\rangle_B \left| v_n \right\rangle_A
\]

**Pure State of AB system**

\[
S_{AB} = 0
\]
Typical-State Condition:

\[ S_{AB} = 0, \quad S_{ABC} = S_C \]

Strong subadditivity:

\[ S_{AB} \geq S_B + S_{ABC} - S_{BC} \]

\[ 0 \geq S_B + S_C - S_{BC} = I(B \parallel C) \geq 0 \]

\[ I(B \parallel C) = 0 \]

No correlation between B and C!
Typical-State Condition of A and B:

\[ S_{AB} = 0 \]

B is almost maximally entangled with (a part of) A!

Mutual information between B and C vanishes.

\[ I(B \parallel C) = 0 \]

No correlation between B and C!
No-Drama Condition across Horizon:

\[ ds^2 = -dt^2 + dx^2 \]

\[ t = \frac{1}{\kappa} \exp(\kappa \sigma_B) \sinh(\kappa \tau), \]
\[ x = \frac{1}{\kappa} \exp(\kappa \sigma_B) \cosh(\kappa \tau) \]

\[ ds^2 = \exp(2 \sigma_B) \left( -d\tau^2 + d\sigma_B^2 \right) \]

\[ t = -\frac{1}{\kappa} \exp(-\kappa \sigma_C) \sinh(\kappa \tau), \]
\[ x = -\frac{1}{\kappa} \exp(-\kappa \sigma_C) \cosh(\kappa \tau) \]

\[ ds^2 = \exp(-2 \sigma_C) \left( -d\tau^2 + d\sigma_C^2 \right) \]
No-Drama Condition across Horizon:

\[ \hat{\phi}(\tau, \sigma) = \Theta(x^- < x_H) \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left[ \hat{a}_\omega^{(B)} \exp(i\omega(\sigma_B - \tau)) + h.c. \right] \]

\[ + \Theta(x^- > x_H) \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left[ \hat{a}_\omega^{(C)} \exp(i\omega(\sigma_C + \tau)) + h.c. \right] \]

\[ \hat{a}_\omega^{(B)} \left| 0 \text{ Rindler} \right\rangle = \hat{a}_\omega^{(C)} \left| 0 \text{ Rindler} \right\rangle = 0 \]

\[ \left| 0_{in} \right\rangle \propto \prod \exp \left[ \exp \left( -\frac{\pi\omega}{\kappa} \right) \hat{a}_\omega^{(B)*} \hat{a}_\omega^{(C)*} \right] \left| 0 \text{ Rindler} \right\rangle \]

Unruh Correlation:

\[ \left| 0_{in} \right\rangle \propto \prod \left[ \sum_n \exp \left( -\frac{n\pi\omega}{\kappa} \right) \left| n \right\rangle_B \left| n \right\rangle_C \right] \]
No-Drama Condition across Horizon:

\[ |\Phi\rangle_{BC} \propto \sum_{n} \exp\left(-\frac{n \pi \omega}{\kappa}\right) |n\rangle_B |n\rangle_C \]

\[ S_{BC} = 0, \quad S_{ABC} = S_A \]

strong subadditivity

\[ S_{BC} \geq S_B + S_{ABC} - S_{AB} \]

\[ 0 \geq S_A + S_B - S_{AB} = I(A \mid\mid B) \geq 0 \]

\[ I(A \mid\mid B) = 0 \]

No correlation between A and B!
No-drama Condition across Horizon:

\[ S_{BC} = 0 \]

\textit{B is highly entangled with C!}

\textit{Mutual information between A and B vanishes.}

\[ I(A \parallel B) = 0 \]

\textit{No correlation between A and B!}
Typical-State Condition of A and B:

\[ |\Psi\rangle_{AB} \Rightarrow S_{AB} = 0 \]
No-Drama Condition across Horizon:

\[ |\Phi\rangle_{BC} \implies S_{BC} = 0 \]

Quantum monogamy
AMPS discard the no-drama condition and propose FIREWALL on the horizon.
2. Canonical Typicality for Non-Vanishing Hamiltonian, and No Firewalls
Problem for Proposition I of Page Curve Hypothesis:

The area law of entanglement entropy is broken in a sense of ordinary many body physics, though outside-horizon energy density in BH evaporation is much less than the Planck scale.
$S_{EE} \propto \frac{\partial A}{\partial \partial B}$ ← standard area law of entanglement entropy

$|\Psi\rangle_{AB} \approx |0\rangle_{AB}$

for low excited states
$|\Psi\rangle_{AB} = \frac{1}{\sqrt{|A|}} \sum_{n=1}^{|A|} |u_n\rangle_A |\tilde{v}_n\rangle_B$

**LLPP Typicality $\Rightarrow$**

**Qubit network model**

$|A| = 2^{V_A}$

$S_{EE} = \ln |A| \propto V_A$

*Not area law, but volume law for highly excited states!*
This is because zero Hamiltonian (complete degeneracy) is assumed in the LLPP theorem. This is also an implicit premise of the Page curve hypothesis.

\[ H_{AB} = 0. \]
In BH physics, we have to treat canonical typicality with non-vanishing $H$ in a precise manner. Then non-maximal entanglement emerges and makes near-horizon regions smooth. Thus no firewalls appear.

M. Hotta and A. Sugita, arXiv:1505.05870
Microcanonical Energy Shell

(not a tensor product of the sub-Hilbert spaces)

\[ H_{AB} \left| E_j \right\rangle = E_j \left| E_j \right\rangle \]

\[ \Delta(E) = \{ j \mid E_j \in [E - \delta, E] \} \]

Microcanonical Energy Shell:

\[ V_{ES}(E) = \left\{ \sum_{j \in \Delta(E)} c_j \left| E_j \right\rangle \right\} \]
\[
\rho_A = Tr_B \left( |\Psi\rangle_{AB} \langle \Psi |_{AB} \right)
\]
\[
= \frac{1}{N_A} \left[ I_A + \sum_{n=1}^{N_A^2-1} \langle T_n \rangle T_n \right]
\]

Bloch Representation of higher-dim quantum states

\[
T_n^\dagger = T_n, \quad Tr[T_n] = 0, \quad Tr[T_n T_n'] = N_A \delta_{nn'}
\]

\[
\langle T_n \rangle = Tr[T_n \rho_A]
\]
Evaluate $\langle T_n \rangle$ for $|\Psi\rangle_{AB} = \sum_{j \in \Delta(E)} c_j |E_j\rangle$.

$D = \dim V_{ES}(E) = \dim \left\{ \sum_{j \in \Delta(E)} c_j |E_j\rangle \right\}$

$N_A \ll N_B$

$D \propto \exp(\gamma V_B(N_B)) \gg 1$

for ordinary systems.
Uniform Ensemble on Microcanonical Energy Shell:

\[ p(c) \propto \delta \left( \sum_{j \in \Delta(E)} |c_j|^2 - 1 \right) \]

\[ \int p(c) d^D p = 1 \]

\[ \bar{f} = \int f(c) p(c) d^D p \]

\[ c_j c_{j}^* = \frac{1}{D} \delta_{jj'}, \]

\[ c_j c_k c_{j'} c_{k'}^* = \frac{1}{D(D+1)} \left( \delta_{jj'} \delta_{kk'} + \delta_{jk'} \delta_{j'k} \right) \]
\[
\left( \langle T_n \rangle - \langle T_n \rangle \right)^2 \leq \frac{\| T_n \|^2}{D + 1}
\]

\[
\text{Max eigenvalue}
\]

\[
\text{Tr}_A \left( \rho_A - \bar{\rho}_A \right)^2 \leq \left( \frac{1}{N_A} \sum_{n=1}^{N_A^2-1} \| T_n \|^2 \right) \frac{1}{D + 1}
\]

\[
N_B \text{ independent!}
\]

\[
D \propto \exp(\gamma V_B) = O(\exp(10^{23})) \gg 1
\]

\[
\| \rho_A - \bar{\rho}_A \| \leq O(\exp(-\gamma V_B))
\]
Hotta-Sugita (2015) as a response to a BH firewall debate with Daniel Harlow

Harlow argued a canonical typicality in a weak interaction limit.

\[ H = H_A + H_B + \cdots \]

Negligibly small

\[ \langle H \rangle = E_A + E_B = \text{const}. \]
Without any proof, Harlow argued these only in the weak interaction limit.
Harlow pointed out a possibility that BH firewalls may exist even after non-zero Hamiltonian is taken account of.

\[ HR = A \cup B, \quad BH = C \]

\[ 1 \ll |A|, |B|, |C|, |B \| C| \ll |A| \]

\[ \rho_{BC} \propto \exp(-\beta (H_B + H_C)) \]
\[ = \exp(-\beta H_B) \otimes \exp(-\beta H_C) \]

No Correlation, just like \( I_B \otimes I_C \)

\[ \text{Tr} \left[ (\partial \varphi(x))^2 \rho_{BC} \right] = \infty ? \]

FIREWALL?

However, the worry is useless. We can prove nonexistence of firewalls for general systems by using the general theory of canonical typicality.


Irrespective of the strength of the interaction between $B$ and $C$, \[
\rho_{BC} \propto \exp\left(-\beta(H_B + H_C + V_{BC})\right)
\]

$|B| \ll |C| \ll |A|$
Actually, a correlation exists between B and C for small interactions.

\[
H_B \quad \quad \quad \quad V_{BC} \quad \quad \quad \quad H_C
\]

\[
\rho_{BC} \propto \exp\left(-\beta(H_B + H_C + V_{BC})\right)
\]

Harlow’s worry: \[
\lim_{V_{BC} \to 0} |Tr[\rho_{BC} V_{BC}]| = \infty!?
\]
**Border shift does not change physics at all.**

\[ H_B + H_C + V_{BC} = H_{B'} + H_{C'} + V_{B'C'} \]

\[ \rho_{BC} = \rho_{B'C'} \propto \exp\left( -\beta(H_{B'} + H_{C'} + V_{B'C'}) \right) \]

Merely an ordinary local operator of C'.

\[ |\text{Tr}\left[\rho_{BC}V_{BC}\right]| = |\text{Tr}\left[\rho_{B'C'}V_{BC}\right]| < \infty \quad \text{No firewall!} \]
**Remark:** for ordinary weakly interacting quantum systems, entanglement entropy is upper bounded by thermal entropy, as long as stable Gibbs states exist.

\[ H_A \quad \text{A} \quad H_B \quad \text{B} \]

\[ |\Psi\rangle_{AB} \]

\[ E_A + E_B = E \]

**Arbitrary state:** \( \rho_A = Tr_B \left[ |\Psi\rangle_{AB} \langle \Psi |_{AB} \right] \)

**Gibbs state:** \( \bar{\rho}_A = \exp(-\beta(E)H_A) / Z_A(\beta(E)) \)

\[ S_{EE} = -Tr[\rho_A \ln \rho_A] \leq -Tr[\bar{\rho}_A \ln \bar{\rho}_A] = S_{\text{thermal}} \]
Conventional “proof”:

\[ I = -\text{Tr}_A[\rho_A \ln \rho_A] - \lambda_1(\text{Tr}_A[\rho_A H_A] - E_A) - \lambda_2(\text{Tr}_A[\rho_A] - 1) \]

\[ \delta I = 0 \]

\[ \tilde{\rho}_A = \exp(-\beta H_A) / Z_A(\beta) \]

\[ -\text{Tr}[\rho_A \ln \rho_A] \leq -\text{Tr}[\tilde{\rho}_A \ln \tilde{\rho}_A] \]

If a stable Gibbs state exists, it attains the maximum of the von Neumann entropy with average energy fixed.
Unfortunately, the typicality argument cannot be applied to Schwarzschild BH evaporation!

Actually, from our result, the typical state must be a Gibbs state, but...
No stable Gibbs state for Schwarzschild BH
due to negative heat capacity! (Hawking –Page, 1983)

\[ \langle E \rangle = M_{BH} = \frac{1}{8\pi GT} \rightarrow \frac{d\langle E \rangle}{dT} = -\frac{1}{8\pi GT^2} < 0 \]

If there exists a stable Gibbs state, heat capacity must be positive.

\[ Z_{BH}(\beta) = Tr[\exp(-\beta H_{BH})] \]

\[ \frac{d\langle E \rangle}{dT} = \frac{\left\langle (E - \langle E \rangle)^2 \right\rangle}{T^2} > 0 \]
Thus, a system of a black hole and Hawking radiation is not in typical states, at least in the sense of the Page curve hypothesis, during BH evaporation. Because we have no stable Gibbs state, “thermal entropy” of Schwarzschild BH ($A/(4G)$) is not needed to be a upper bound of entanglement entropy.
In ordinary quantum systems, \( |\Psi\rangle_{AB} \) is a typical state with almost certainty after a relaxation time. 

\[ E_{\text{total}} = \text{const}. \]

Microcanonical Energy Shell
The state of BH evaporation can be non-typical until the last burst.

\[ U_{BH} \otimes I_{HR} \]

Fast scrambling of BH does not contribute to entanglement between BH and HR.

\[ U^{( \text{emission} )} \]

Non-chaotic HR emission generated by smooth space time curvature outside horizon.

Sub-Hilbert space of non-typical states
If so, how is the Page curve modified?
The moving mirror model is totally unitary. So we are able to learn how the information can be retrieved.

The model is a tool to explore the Page curve hypothesis and its modification by using various mirror trajectories.
Page Curve in Moving Mirror Model

Mirror Trajectory:

\[ x^+ = -\ln \left[ \frac{1 + \exp(-\kappa x^-)}{1 + \exp(\kappa(x^- - h))} \right] \]
Entanglement Entropy of Emitted Radiation

\[ \rho_A = Tr_B \left[ \Psi_{AB} \langle \Psi \mid_{AB} \right] \]

\[ S_{EE} = -Tr\left[ \rho_A \ln \rho_A \right] \]
Entanglement Entropy between radiation and its compliment:

\[
S_{EE} = \frac{1}{12} \ln \left( \frac{\left( f(x_2^-) - f(x_1^-) \right)^2}{\partial_- f(x_2^-) \partial_- f(x_1^-) \varepsilon_2^- \varepsilon_1^-} \right)
\]

(Holzhey-Larsen-Wilczek)
Renormalized Entanglement Entropy

$$\Delta S_{EE} = S_{EE} - S_{EE} \mid_{vac}$$

Holzhey- Larsen-Wilczek
Page Curve in Moving Mirror Model

$\Delta S_{EE}$

Page time

$\kappa = 1, h = 500, x_1^- = -2$

Thanks to Daniel Harlow
In order to reproduce the Page curve, very strange time evolution induced by nonlocality is required for the mirror trajectories!

\[
| \Psi \rangle_{BH} \otimes | 0 \rangle_{HR}
\]

Quite different time schedules of information leakage for black holes with the same mass.
Possible modification of the Page curve, assuming local dynamics.

Quantum Gravity

$x^+ = -\ln \left[ \frac{1 + \exp(-\kappa x^-)}{1 + \exp(\lambda(x^- - h))} \right]$

Planck-energy last burst with a tiny amount of information

Hawking Radiation

Mirror Trajectory:
The entangled partner of the Hawking particle is zero-point fluctuation with zero energy. (Wilczek, Hotta-Schützhold-Unruh)
Entangled Partner

Particle A

Particle B

cf. “Hawking Theorem” of Mathur

Entanglement

Hawking Particle

Zero-Point Fluctuation Flow with Zero Energy

(Wilczek, Hotta-Schützhold-Unruh, Hawking)
Modified Page Curve in Moving Mirror Model

\[ \kappa = 1, \lambda = 100, h = 500, x_1^- = -2 \]

contribution of zero-point fluctuation without energy cost for information storage

All of the information comes out at the end by zero-point fluctuation flow.
(More Sharper) Strong Subadditivity “Paradox”

Strong subadditivity:

\[ S_{AB} \geq S_B + S_{ABC} - S_{BC} \]

No Drama:

\[ S_{BC} = 0, \quad S_{ABC} = S_A \]

\[ S_{AB} \geq S_B + S_A \]

\[ S_A > S_{AB} \]
(More Sharper) Strong Subadditivity “Paradox”

Page Curve Hypothesis

\[ S_A > S_{AB} \]

\[ S_A > S_{AB} \geq S_B + S_A \]

\[ 0 > S_B \]
Late radiation

Early radiation

Remnant & Zero-Point Fluctuation Flow

\[ S_A < S_{AB} \]

until the last burst.

Thus, no strong subadditivity paradox!
We don’t care the no drama condition breaks at the last burst, because the horizon is affected by quantum gravity.
Last Comment

about Strong Subadditivity Paradox

\[ S_{BC} = 0 \text{ is not a precise expression of the no drama condition.} \]

M. Hotta, J. Matsumoto and K. Funo, 2014
In order to keep strict locality of subsystems for strong subadditivity, the Hawking radiation needs zero-point fluctuation flows to be in a pure state.

\[ \rho_{BC'} = \text{Tr}_{V_{past}} \rho_{AV_{future}} \left[ \left| \Psi \right\rangle \langle \Psi \right| \right] \]

\[ S_{BC'} > 0 \]

\[ S_{BC} > 0 \]

“The strong subadditivity paradox” is a superficial one, anyway.
Frequently Made Mistakes by String Physicists

Somebody says, “Hawking radiation comes from the horizon.”

Hawking particle
Frequently Made Mistakes by String Physicists

The correct answer: Hawking radiation does not become real particles on the horizon!
Frequently Made Mistakes by String Physicists

Somebody says, “Pair-Created particles are entangled, of course.”

- Hawking particle with positive energy
- Negative energy flux
- $r_{BH}$
Frequently Made Mistakes by String Physicists

The correct answer:
“Pair-Created” particles are not entangled!
Black hole formation via gravitational collapse
Summary of Part I

- Adopting canonical typicality for nondegenerate systems with nonvanishing Hamiltonians, the entanglement becomes non-maximal, and BH firewalls do not emerge.

- Typical states must be Gibbs states for smaller quantum systems. If we have stable Gibbs states for old Schwarzschild BH’s (and small AdS BH’s), the heat capacity must be positive. Because it is actually negative, the states of BH evaporation are not typical.

⇒ Inevitable Modification of the Page Curve

Note: for a large AdS BH and Hawking radiation in a thermal equilibrium, the entanglement entropy equals the thermal entropy of the smaller system.
Part II

ZERO-POINT FLUCTUATION AND BLACK HOLE ENTROPY
CHAPTER 1
QUANTUM ENERGY TELEPORTATION
I would like to speak an interesting feature of quantum energy-momentum tensor. Though the operators are local, quantum energy itself is a quite nonlocal concept from an operational viewpoint of quantum information.

\[ \left[ \hat{T}_{\mu\nu}(x), \hat{T}_{\mu\nu}(y) \right] = 0, (x \neq y) \]

Locality of Operators

\[ \langle \Psi | \hat{T}_{\mu\nu}(x) | \Psi \rangle \]

Nonlocal
Performing a distant measurement of vacuum fluctuation, the zero-point energy becomes activated and can be extracted by local operation dependent on the measurement result. This protocol is called quantum energy teleportation (QET). This provides a new method of entropy decreasing of black holes.
In order to grasp QET, let us first discuss a massless scalar field in 1+1 dimensional Minkowski spacetime.

\[
\left[ \partial_t^2 - \partial_x^2 \right] \phi(t, x) = 0
\]

\[x^\pm = t \pm x\]

\[\partial_+ \partial_- \phi = 0\]

\[\phi = \phi_R(x^-) + \phi_L(x^+)\]

Right-mover component

Left-mover component
Chiral Momentum Operators

\[ \Pi_{\pm}(x) = \Pi(x) \pm \partial_x \phi(x) \]

Primary degrees of freedom for left- and right-mover modes of field

\[ \exp(itH)\Pi_{\pm}(x)\exp(-itH) = \Pi_{\pm}(x \pm t) = \Pi_{\pm}(\pm x^\pm) \]

Energy-Momentum Tensor

\[ T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial_\lambda \phi \partial^{\lambda} \phi) \]
Energy Density Operator

\[ \varepsilon(x) = T_{tt}(x) \]

\[ = \frac{1}{2} \Pi(x)^2 + \frac{1}{2} (\partial_x \phi(x))^2 - \varepsilon_o \]

\[ = \frac{1}{4} \Pi_+(x)^2 + \frac{1}{4} \Pi_-(x)^2 - \varepsilon'_o \]

Hamiltonian

\[ H = \int_{-\infty}^{\infty} \varepsilon(t, x) dx \]

Vacuum State

\[ H|0\rangle = 0 \]

zero-point energy subtraction
PREPARATION: GENERAL MEASUREMENT

(POVM Measurement, Indirect Measurement)

The probe measurement result $\alpha$ includes information about the system!!

Ideal Measurement $\Rightarrow$ $U_m |\psi_S\rangle |0_P\rangle = \sum_n c_n |n_S\rangle |n_P\rangle$ Perfect correlation between S and P.

The probe measurement result $\alpha$ includes information about the system!!
Measurement Operator for $S$

$$M_\alpha = \langle \alpha_P | U_m | 0_P \rangle$$

time-evolution operator of measurement interaction between system and probe

$$\text{Tr}_P \left[ \left( I_S \otimes | \alpha \rangle \langle \alpha |_P \right) U_m \left( \rho_S \otimes | 0 \rangle \langle 0 |_P \right) U_m^\dagger \right] = M_\alpha \rho_S M_\alpha^\dagger$$

projection measurement of probe

initial state

post-measurement state of system

Positive Operator Valued Measure (POVM)

$$\begin{cases} M_\alpha^\dagger M_\alpha | \sum_\alpha M_\alpha^\dagger M_\alpha = I \end{cases}$$

$$M_\alpha^\dagger \neq M_\alpha, \quad M_\alpha^2 \neq M_\alpha$$

Emergence probability of $\alpha$

$$p_\alpha = \text{Tr} \left| \psi \langle \psi | M_\alpha^\dagger M_\alpha \right|$$

Post-measurement state

$$| \psi_\alpha \rangle = \frac{M_\alpha | \psi \rangle}{\sqrt{\langle \psi | M_\alpha^\dagger M_\alpha | \psi \rangle}}$$

General measurements are not always project measurements of the system.
Let us consider a two-level spin which stays at $x = x_A$ as the probe system in order to detect local fluctuation of $\phi$.

**Measurement Model:** Instantaneous Interaction between Field and $\frac{1}{2}$ Spin

The initial state of the spin is the up state of the z component. After the measurement interaction, the z component of the spin is measured.

**Measurement Operation:**

$$U_m = \exp\left[-i(\Phi_S \otimes \sigma^y_P)\right]$$

$$\Phi_S = \frac{\pi}{4} - \int_{x_1}^{x_2} \lambda_A(x) \Pi_+(x) dx$$

**Measurement Operators:**

$$M_A(\pm) \rho_S M_A^\dagger(\pm) = \text{Tr}_P \left[(I \otimes |\pm\rangle\langle\pm|_P)U_m(\rho_S \otimes |+\rangle\langle+|_P)U_m^\dagger\right]$$

$$M_A(+) = \cos \Phi_S, \quad M_A(-) = \sin \Phi_S$$
Information Flow via the Interaction

Emergence probability of $\alpha = \pm$ for the vacuum state

$$p_\alpha = \text{Tr} \left[ 0 \langle 0 | M_A(\alpha)^\dagger M_A(\alpha) \right]$$

$$M_A(+) = \cos \Phi_S, \quad M_A(-) = \sin \Phi_S$$

$$\Phi_S = \frac{\pi}{4} - \int_{-\infty}^{\infty} \lambda_A(x) \Pi_+(x) dx$$

We obtain the same probability for $\alpha$:

$$p_\alpha = \frac{1}{2}$$

1-bit information about the quantum fluctuation of the field
Post-Measurement States of Quantum Field

\[ \alpha = + \Rightarrow |\psi_+\rangle = \frac{1}{\sqrt{2}} \left( e^{\frac{i\pi}{4}} |\lambda_A\rangle + e^{-\frac{i\pi}{4}} |\lambda_A\rangle \right) \]

\[ \alpha = - \Rightarrow |\psi_-\rangle = \frac{1}{\sqrt{2}} \left( e^{\frac{-i\pi}{4}} |\lambda_A\rangle + e^{\frac{i\pi}{4}} |\lambda_A\rangle \right) \]

Left-mover coherent state of field

\[ |\pm \lambda_A\rangle \propto \exp \left[ \pm i \int_{x_1}^{x_2} \lambda_A(x) \Pi_+(x)dx \right] |0\rangle \]

\[ \langle \psi_+ | \psi_- \rangle \neq 0 \quad \leftarrow \text{General measurement} \]
Average post-measurement state:

\[
\rho' = \sum_{\alpha=\pm} M_A(\alpha) \langle 0 | M_A^\dagger(\alpha) = \sum_{\alpha=\pm} p_\alpha |\psi_\alpha\rangle \langle \psi_\alpha |
\]

Average excitation energy injected by measurement device

\[
E_A = \text{Tr}[\rho' H] = \sum_{\alpha=\pm} \langle 0 | M_A^\dagger(\alpha) H M_A(\alpha) | 0 \rangle
\]

This energy is calculated as positive:

\[
E_A = \int_{-\infty}^{\infty} \left( \partial_x \lambda_A(x) \right)^2 dx > 0
\]
This excitation energy is locally distributed around $X = X_A$ soon after the measurement.

However, it is proven that this energy cannot be completely withdrawn from the field by arbitrary local operations around $X = X_A$. 
Quantum measurements decrease entanglement between quantum fluctuation of the field around $x = x_A$ and quantum fluctuation in distant regions.

$$\text{ENT}_{AB}(\rho_m) < \text{ENT}_{AB}(|0\rangle \langle 0|)$$

Extraction of Information about Field

Measurement of Local Fluctuation of Field

Weak entanglement entanglement
Because no local operations around $x = x_A$ increase the entanglement, the zero-energy state (the vacuum state) cannot be recovered. Hence, we have residual energy of the field in local cooling processes.

\[
\text{ENT}_{AB} = \text{ENT}_{A} \left[ \rho_A (1) \right] = \rho_{m} \text{ENT}_{AB} \left( \rho_{m} \right) \neq \Gamma \rho_{m} \text{ENT}_{AB} \left( \rho_{m} \right) = 0
\]

Entanglement gap

\[
\Gamma_A [\rho_m] \neq |0\rangle \langle 0| \Rightarrow \text{Tr} [H \Gamma_A [\rho_m]] > \langle 0 | H | 0 \rangle = 0
\]

Residual Energy

Local Operation

Weak entanglement
A part of the residual energy can be extracted operationally by Bob, who stays at a distant point and knows the result of the general measurement. This is Quantum Energy Teleportation (QET).
Firstly, at time $t=0$, we perform the same measurement previously discussed. Then, the measurement device excites the left-mover mode with energy $+E_A$.

$$\langle \varepsilon(x) \rangle$$

$$\Phi = -\Phi = +$$

$$\int + \Pi - = \Phi$$

$$M_A(\alpha) = \cos \Phi_S, \quad M_A(-) = \sin \Phi_S$$

$$\Phi_S = \frac{\pi}{4} - \int_{x_1}^{x_2} \lambda_A(x) \Pi_+(x) dx$$

STEP 1
Time Evolution of Post-Measurement State

\[
\rho_\alpha(t) = U(t) \frac{M_A(\alpha) |0 \rangle \langle 0 | M_A(\alpha)^\dagger}{\langle 0 | M_A(\alpha)^\dagger M_A(\alpha) | 0 \rangle} U(t)^\dagger
\]

\[
= U(t) |\psi_\alpha \rangle \langle \psi_\alpha | U(t)^\dagger
\]

\[
(U(t) = \exp[-itH])
\]

In this model, energy density and its time evolution are independent of the measurement result:

\[
\text{Tr}[\rho_\alpha(t) \varepsilon(x)] = \left( \partial_x \lambda_A(x + t) \right)^2
\]
Next, at time $t=T$, the measurement result is announced to Bob at $x = x_B$ and a local operation dependent on the measurement result is performed.
Bob’s Local Operation dependent on Measurement Results

\[ U_B(\alpha) = \exp\left[ i \alpha g \int_{-\infty}^{\infty} p_B(x) \Pi_+(x) dx \right] \]

- Measurement result
- Localized function around Bob
- \( g \) is fixed so as to extract maximum energy for the field.
Finally, positive energy is extracted by the operation with generation of negative-energy left-mover excitation of the field.

\[ U_B(\alpha)|\psi_\alpha\rangle \]

Positive Energy Release from Field with Simultaneous Generation of Negative-Energy Wavepacket

\[ \langle \varepsilon(x) \rangle \]

\[ t > T \]

Negative Energy Excitation

\[ + E_B \]

\[ - E_B \]

STEP 3
\[
\rho = \sum_{\alpha = \pm} U_B(\alpha) U(T) M_A(\alpha) |0\rangle \langle 0| M_A^\dagger(\alpha) U^\dagger(T) U_B^\dagger(\alpha)
\]

Teleported Energy from Alice to Bob:

\[
E_B = \frac{4 |\langle 2\lambda_A | 0 \rangle|^2}{\pi \int_{-\infty}^{\infty} p_B(x)^2 \, dx \left[ \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy p_B(x) \frac{1}{(x - y + T)^2} \lambda_A(y) \right]^2}
\]
The amount of extracted energy by Bob is upper-bounded by the residual energy of the local cooling by Alice!!

$$\min_{\Gamma_A} E_A \geq E_B$$
Remark: a part of the energy injected by Alice can be extracted from other regions.

\[ \langle \mathcal{E}(x) \rangle \]

\[ \min_{\Gamma_A} E_A \]

\[ \alpha = \pm \]

\[ \sum_{n \in \Gamma} E_{B_n} \]

Teleported energy is hidden like oil fields!

\[ \min_{\Gamma_A} E_A \geq \sum_{n} E_{B_n} \]
Spacetime Diagram of QET

Similar to Generation of Hawking Radiation Outside Black Hole Horizon
The physical process so far known to decrease the horizon area is the emission of Hawking radiation. However, QET is also capable of decreasing the horizon area again!
Decrease of Horizon Area

\[ \Delta E = E_A - E_B + M_A(\alpha) - M_B(\alpha) \]
Controlled Hawking Process by QET


Black Hole

Measurement Information

1bit

Nontrivial Relation with Black-Hole Entropy!

Initial black hole entropy

\[ S_{BH} = \frac{1}{4G} A = 4\pi GM_{BH}^2 \]

If the one-bit information is lost, the horizon area cannot decrease via QET!
CHAPTER 2
INFORMATIONAL COSMIC CENSORSHIP CONJECTURE
“Another Firewall Paradox”

The point is **Reeh-Schlieder theorem** in quantum field theory.

**In other words,**

**Entanglement of Zero-Point Fluctuation**
Reeh-Schlieder theorem: The set of states generated from $|0_{in}\rangle$ by the polynomial algebra of local operators in any bounded spacetime region is dense in the total Hilbert space of the field. Thus, in principle, any state can be arbitrarily closely reproduced by acting a polynomial of local operators of $E'$ on $|0_{in}\rangle$.

\[
\forall |\Psi\rangle \approx \sum_n \int_{E'} a_n(x_1, \cdots, x_n) \hat{O}_1(x_1) \cdots \hat{O}_n(x_n) d^n x |0_{in}\rangle
\]
Note that the moving mirror models mimic gravitational collapse to black holes and reproduce Hawking radiation.
$x^+ = f(x^-)$

$\Phi_{out}(x^-)$

$\Phi_{in}(x^+)$

$|0_{in}\rangle$
Mirror Trajectory:

\[ x^+ = f(x^-) \quad (x^\pm = t \pm x) \]

Boundary Condition:

\[ \hat{\phi} \bigg|_{x^+ = f(x^-)} = 0 \]

Solution:

\[ \hat{\phi}(x, t) = \hat{\phi}_{in}(x^+) - \hat{\phi}_{in}(f(x^-)) \]

Scattering Relation:

\[ \hat{\phi}_{out}(x^-) = \hat{\phi}_{in}(f(x^-)) \]
Out-going energy flux: \[ \hat{T}_{--} =: \partial_- \hat{\phi} \partial_- \hat{\phi} := \partial_- \hat{\phi}_{out} \partial_- \hat{\phi}_{out} : \]

\[
\langle 0_{in} | \hat{T}_{--}(x^-) | 0_{in} \rangle = -\frac{1}{24\pi} \left[ \frac{\partial_-^3 f(x^-)}{\partial_- f(x^-)} - \frac{3}{2} \left( \frac{\partial_-^2 f(x^-)}{\partial_- f(x^-)} \right)^2 \right]
\]

derived from \[ \hat{\phi}_{out}(x^-) = \hat{\phi}_{in}(f(x^-)) \]

**The moving mirror emits radiation!**

*(Dynamical Casimir Effect)*
Moving Mirror Model in 1+1 dim. mimics 3+1 dim. spherical gravitational collapse.

\[ x^+ = f(x^-) = -\frac{1}{\kappa} \ln(1 + e^{-\kappa x^-}) \]

\[ f(x^- \approx -\infty) \approx x^- \quad \text{The mirror does not move in the past.} \]

\[ f(x^- \approx \infty) \approx -\frac{1}{\kappa} \exp(-\kappa x^-) \quad \text{The mirror accelerates and approaches the light trajectory,} \quad x^+ = 0. \]
\[ x^+ = f(x^-) \]

\[ x^+ = 0 \]

\[ x^- \quad x^+ \]
The mirror emits thermal flux in the late time.

\[ \langle 0_{in} | \hat{T}_{-\infty} (x^- >> 1 / \kappa) | 0_{in} \rangle = \frac{\pi}{12} T^2 \]

**Temperature:**  
\[ T = \frac{\kappa}{2 \pi} \]

\[ \langle 0_{in} | \hat{a}^{(out)}_\omega \dagger \hat{a}^{(out)}_\omega | 0_{in} \rangle \propto \frac{1}{\exp \left( \frac{2\pi}{\kappa} \omega \right) - 1} \]

**Hawking Radiation!**
Remark: if BH evaporates satisfying unitarity, its Penrose diagram is similar to that of the moving mirror model in which the mirror will stop in the future.

S. Hawking, 2014
Note that the Reeh-Schlieder property is maintained in the time evolution:
\[ \left| 0_{\text{in}} \right\rangle \rightarrow \left| f' \right\rangle. \]

Even in the future infinity, we may remotely generate any excitation with some probability smaller than 1.

\[ \hat{O}_{E} = \sum_{n} \int_{E} a_{n}(x_{1}, \ldots, x_{n})\hat{O}_{1}(x_{1})\cdots\hat{O}_{n}(x_{n})d^{n}x \]
Firewall Measurement Paradox:

Imagine that, besides the background Hawking radiation, a wave packet with positive energy of the order of the radiation temperature appears at $x^- = x_{fw}$. Then the firewall (FW) appears at $x^+ = g_h(x_{fw})$.

If measurement operator is constructed from Reeh-Schlieder operation, an arbitrary post-measurement state including firewalls can emerge.

$$\hat{M}_{iE} \propto \hat{O}_E$$

$$\left| f^\prime \right\rangle = \sum_i \left| \psi_i \right\rangle_E \left| i \right\rangle_L$$
Because the mirror merely stretches the modes of the field, the future measurement is equivalent to a past measurement for the in-vacuum state.

\[ \hat{M}_{iE} \iff \hat{M}_{iE'} \]

We can analyze the problem using past infinity.
Informational Cosmic Censorship Conjecture

“Resolution of the Paradox from a viewpoint of Quantum Measurement Energy Cost”

The local measurements generally inject energy on average to the system in $|0_{in}\rangle$ owing to its passivity property (Pusz and Woronowicz). Thus the measurements always require an energy cost.

Though the Reeh-Schlieder theorem is mathematically correct, it does not guarantee that the measurement energy to create $|i_L\rangle$ is finite.
If $\hat{M}_{iE}^\dagger \hat{M}_{iE}$ is regular, no outstanding peak of energy flux appears.

The two-point correlation functions for non-singular measurements simply obey a power-law decay as a function of the distance. $\Rightarrow$ No Firewalls!

If we assume FW appears at \( x^+ = g_h(x_{fw}) \) with less-than-1 probability in a finite-energy measurement, then

\[
E_{fw} < O\left(\frac{1}{12\pi l}\right) \ll E_{\text{planck}} \Rightarrow \text{No Firewall appears!}
\]

\[
E_{fw} \delta(x^+ - g_h(x_{fw}))
\]

Energy cost of measurement: 

\[ E_+ \geq \frac{rE_{fw}}{1 - 12\pi rlE_{fw}} \]

\[ E_{fw} \rightarrow \frac{1}{12\pi rl} \]

\[ E_+ \rightarrow \infty \]

Energy cost of FW measurement diverges!

Taking account of back reaction, a black hole may be formed in the measurement region during the preparation of huge energy for the firewall measurement.

$\hat{M}_{iE}$

Event Horizon

Huge amount of energy for firewall measurement

$\Rightarrow$ Firewall Information Censorship

Informational extension of Cosmic Censorship

Summary

- **Strong subadditivity paradox** is a superficial one. No firewall is required by the entanglement monogamy.

- **Zero-point fluctuation** can be the purification partner of HR.

- By measuring zero-point fluctuation outside horizon, BH entropy can decrease via quantum energy teleportation.

- **Reeh-Schlieder theorem** poses a measurement-based firewall paradox. However, the amount of measurement energy of firewalls becomes divergent. The effect may cause formation of a new black hole in the measurement region and enclose the measurement device within the event horizon before it outputs results. ⇒ Quantum Information Cosmic Censorship Conjecture